Chapter III

CHRISTIAN HUYGENS'
CONTACT CONE



INTRODUCTION

While it served as a model for most of *Descartes*' erudite contemporaries, his theory of reasoning appeared rapidly insufficient to take account of the multiple experimental discoveries of the second half of the 17th Century. This was because it depended mainly on analogies as its starting point.

And, it was *Christian Huygens* (1629-1695), early on in his scientific career, which questioned the soundness of certain *Cartesian* writings and outlined in his future treatises on paper, particularly with the publication of a treatise on refraction that was innovative in numerous aspects. He was very keen to give to his discoveries a completed formulation, and was not to publish his results until after he had reflected on them for many long years, and he intended that some were not even going to appear until after he had died.



1 - Source Documents

(Figures 3-1)

TRACTATUS DE REFRACT. ET TELESC. LIBER II. 1653.

(PROPOSITIO XI.] ')

Theorema 1).

Si loco conspicilli duarum lentium ejusmodi adaptetur ex solido materiæ diaphanæ frusto, cujus altera superficies convexa sit altera cava, eddem proportione visibilia augebit longinqua, atque conspicillum duarum lentium. Scilicit augmenti ratio ea erit, quæ distantiæ superficiei convexæ à foco suo ad distantiam foci à cava, cui oculus admotus est.

Esto talis specilli continui superficies convexa AM [Fig. 38], ex sphæra cujus N centrum. Superficies vero BQ cava centro P. Et socus superficiei AM seu concursus parallelorum sit G punctum. at R punctum dispersus superficiei BQ radiorum parallelorum qui intra solidum feruntur 1). Porro visibile longinquum sit DED. Itaque ostendendum cum oculus superficiei B applicabitur ea proportione visibile DED augeri, quam habet AG ad GB 4).

Figure 3 - 1

Christian Huygens - Compilation of Huygens works in Latin language: "Propositio XI, Tractatus de refract. Et telesc. Liber II - Proposition XI, Tractatus de Refract. et Telec. Liber II, 1653".

(p. 225 in : Œuvres complètes de Christiaan Huygens en 20 volumes (1888 - 1950), tome XIII, fasc.1, Dioptrica I, (1653; 1666), Hollandsche Maatschappig van Wetenschappen, Den Haag, Martinus Nijhoff, 1916).

It's in the compilation of *Huygens* works, edited after 1905 by the Dutch Society of Sciences, that we are to find in pages 224-229 of the second part of volume XIII, the "Tractatus de refract. et telesc. Liber II. Proposition IX" (Treatise on refraction and telescopes), written in 1653, which makes the commentary On Discours VII of Descartes' "The means of perfecting vision" from his "La Dioptrique". The text, published in Latin, includes a figure in Huygens' hand. The text has been translated into French by the compilers of the "Complete works of Christian Huygens".

I have made an analysis of this document in summary form and will discuss noteworthy points that

are historically interesting in regard to neutralization of the corneal diopter, contact lenses, and contact devices. Firstly, this text represents a critical complement for the theme of the contact cone described by *Descartes* in the *Discours septième* analyzed in the previous chapter, and secondly, it includes terminology, which could lead to confusion with the definition of a contact lens.

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1.1 - DESCARTES' DESCRIPTION IS FLAWED

The aspects described by *Huygens* in *Proposition XI* of his "*Treatise on refraction and telescopes*" have their origin in the theory of magnification of images by telescopic devices described by *Descartes* in the *Discours VII* of his *La Dioptrique* called "*The means of perfecting vision*".

Descartes' theory, based on the examination of the case where the tube would be entirely filled with a homogeneous transparent substance, i.e. water or glass, and closed off by two curved surfaces, with the eye being places in front of one of these, was able to satisfy philosophers as an example of a new method of reasoning, but left scientists unsatisfied because the geometric basis and mathematical reasoning were vague and flawed.

1.2 - The Arguments of Huygens

Huygens presents his case in the form of a rigorous demonstration, in five phases:

- description of the problem in § 1,
- enumeration of data in § 2,
- presentation in § 3,
- conclusion in § 4,
- criticism of Descartes' Discourse seventh.

After this presentation, *Huygens* introduces two paragraphs of criticism of *Descartes*' theories.

1.2.1 - Description of the Problem (§ 1)

Huygens places in doubt the veracity of *Descartes*' demonstration explaining the magnification produced by the telescope beginning with a filled transparent tube. He wonders if a transparent body, having both a concave and a convex surface, placed against the eye would be capable of enlarging the image of a distant object in the same way as a Galilean telescope spectacle, as *Descartes* claimed.

One adapts a Cone to the Eye

(*Figure 3 – 2*)

Huygens describes the cone in the following passage:

"One adapts to the eye a body made out of solid and transparent material, and one which possesses both a convex and a concave surface."

"adaptetur ex solido materiae diaphanae frusto, cujus altera superficies convexa fit altera cava"

These terms are not specific for a glass cone and could correspond to the definition of a contact lens of which the solid and transparent material presents effectively a convex and a concave surface. The use of the expression "adapts to the eye" evokes immediately the connection of such a lens with the cornea. This was not *Huygens*' intention, for, in the follow-up to his description he explains that he is investigating the size of the image as a function of the convex surface and of the concave surface "placed against the eye".

PROPOSITION XI 1).

Théorème 2).



Si, au lieu de prendre une lunette composée de deux lentilles, l'une convexe et l'autre concave, on adapte à l'oeil un corps construit d'une matière solide et transparente et possédant une surface convexe et une surface concave, ce corps agrandira les objets lointains dans la même proportion que la lunette composée de deux lentilles. C'est-à-dire, le rapport de la grandeur apparente de l'image à celle de l'objet sera égal au rapport de la distance socale de la surface convexe à la distance du soyer de cette surface à la surface concave, contre laquelle se trouve l'oeil.

Soit AM [Fig. 38] la surface convexe d'une lunette de ce genre, construite d'une pièce, et N le centre de courbure de cette surface. Soit en outre BQ la surface concave, et P son centre de courbure. Supposons que G soit le soyer de la surface AM, c'est-à-dire le point de concours de rayons incidents parallèles, et R le point de dispersion de la surface BQ pour des rayons parallèles se mouvant à l'intérieur du corps solide. Soit de plus DED un objet visible situé à grande distance. Nous devons donc démontrer que lorsque l'oeil est appliqué à la surface B, l'objet DED est augmenté dans le rapport AG: GB 4).

Supposons d'abord que l'oeil, situé en C, ne soit pas encore proche de la surface BQ et construisons une quatrième proportionnelle CK à CR, CP et CB, d'après la prop. XII 5). Comme alors des rayons partis du point C correspondraient au point K après avoir été réfractés à la surface BQ, réciproquement les rayons qui, à l'intérieur du corps transparent solide, se dirigent vers le point K, correspondront au point C après avoir été réfractés à la surface B. De la même manière il arrivera, si l'on construit une quatrième proportionnelle KS aux trois lon-

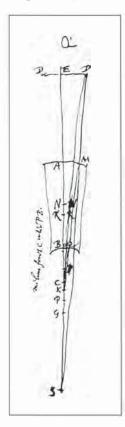
Figure 3 - 2

Christian Huygens - Compilation of Huygens works in Latin language: "Proposition XI, Traité de la Réfraction et des Télescopes, Livre II. 1653".

(p. 224 in : Œuvres complètes de Christiaan Huygens en 20 volumes (1888 - 1950), tome XIII, fasc. I, Dioptrica I, (1653; 1666), Hollandsche Maatschappig van Wetenschappen, Den Haag, Martinus Nijhoff, 1916).

1.2.2 - The Placement of the Components (§ 2)

(Figure 3.-3)



The components used for the demonstration are:

- "A device of this type, constructed in one piece", with one surface convex and another concave, with defined focalities,
- an "object visible at long distance",

The demonstration has to prove that, when the eye is applied to the concave surface, one sees the object to be magnified as defined in the description.

Figure 3 - 3

Christian Huygens - Traité de la refraction et des telescopes. Livre II. Proposition XI. 1653. Fig. 38, p. 226. Illustration of Huygens' demonstration of a telescopic cone, and on the absurdity of Descartes' explanation.

1.2.3 - The Demonstration (§ 3)

The description presented by *Huygens* is long and complex. It envisages two hypotheses: the first of an eye distant from the concave surface and the second of an eye close to it. In the two cases, the relationship of the magnification is independent of the distance between the eye and the concave lens of the system. The magnification is uniquely dependent on the convex lens at the extremity of the tube.

This leaves one to understand that the demonstration of *Descartes' Discourse seventh*, in which he applies the tube directly onto the eye, was erroneous, as the same effect is obtained when it is separated from the eye. *Huygens* admitted later that his demonstration was too complex and, in an annotation, he proposed to himself to simplify the demonstration by limiting it to the case of distant vision.

1.2.4 - The Conclusion (§ 4)

Huygens concludes that, provided that the surface of the tube oriented towards the eye has a smooth surface, be it concave, plano or even convex, the magnification would be the same. It is equal in relationship to the focal distance of the convex surface to the distance from the focus of the eye.

1.2.5 - The Critique of Descartes' Discourse seventii (§ 5)

Finally, Huygens claims:

A - The Concave Surface of the Tube does not produce the Magnification:

"These conclusions are not at all in agreement with the theory by which Descartes tries to explain the invention of the telescope in leading us to consider a solid tubular body of this configuration."

"Hisce vero nequaquam consentiunt ea quibus Cartesius Telescopij inventum explicare contendit, similem huic tubum proponens solidum."

To claim, as *Descartes* does in the *Discourse seventh*, that the magnification is produced by the concave surface of the ocular of the tube or the cone is erroneous according to the demonstration that *Huygens* presented in paragraph 3.

B - The Crossing of the Rays Described by Descartes is Absurd:

"There is also that absurdity in Descartes' explanation: he says that all objects are seen as enlarged because the rays [...] cross at the exterior convex surface of the tubular body."

"Posso illud quoque in eadem Cartesij explicatione absurdum, quod eam ob causam majora omnia videri ait, quoniam ex diversis rei visae punctis venientes radij decussentur in exteriori convexa tubi superficie [...]."

In reality, *Descartes* describes that, by the apposition of the tube to the eye, the crossing of the rays, which occurred at "the entry to the eye" will "occur from the entrance to the tube". Huygens demonstrated above in paragraph 3 that this was not the case and concluded that *Descartes*' explanation is "absurd".

However, as with *Descartes, Huygens* defined neither the length of the tube nor the dimensions of the concave and convex extremities of the tube.

2 - DISCUSSION

Huygens' Work in the Context of the Knowledge of his Epoch

2.1 - Huygens and the Theory of Light

(Figure 3 - 3)

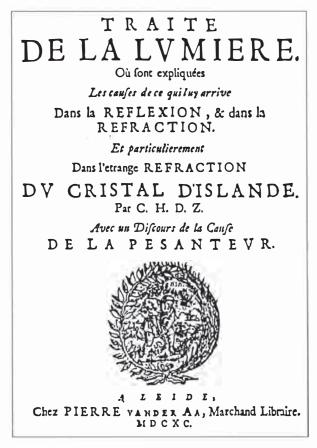


Figure 3 - 3

Christian Huygens - Frontispiece of the "Traité de la lumière" (Treatise on light) and the "Discours de la cause de la pesanteur" (Discourse on the cause of Gravity).

In the *Traité de la Lumière* (Treatise on Light), published in Leyden in 1690, Huygens compares light with sound and, as a result, equates light with the propagation in time of a longitudinal vibration. The *Discours de la cause de la Pesanteur* (Discourse on the cause of Gravity) had been the subject of the general research program of the French Royal Academy of Sciences, and was edited, starting with manuscripts introduced by Huygens from Paris.

The new dissemination of knowledge, that came into existence with the writings of *Kepler* and *Descartes*, liberated the analysis of light from that of visual sensation and its representation in the mind and made light an independent research subject with its own rules.

The theories of light in the first half of the 17th Century are associated with the construction of mechanical models which attempt to answer the question, namely how to explain with the aid of physics the known properties of light, e.g. the rectilinear propagation and reflection of light, refraction and color vision (1). It is this mechanistic aspect and reliance on a thought process largely supporting the analogies of reflection and refraction which Descartes takes account. On the other hand, as indicated above, this Cartesian theory appears to be rapidly becoming inadequate in taking account of the many experimental discoveries of the second half of the 17th Century.

Following *Cartesian* theories, *Huygens* accepts the hypothesis that light is a form of pressure, propagated by high-speed waves across little particles of which the ether is composed. He tries to deduce this theory from the principles of mechanics and the known properties of reflection, and refraction of light. *Huygens* is to defend his hypothesis in the "*Traité de la lumière*" (*Treatise on Light*, 1690). *Huygens* is therefore opposed to *Newton*, who deduced

his own laws for the properties of light starting from his own experiments and observations. For *Newton*, light was a pure substance filling space and composed of different rays, as shown by the deviation of a light ray through a glass prism.

2.2 - HUYGENS AND THE FRENCH ROYAL ACADEMY OF SCIENCES

From the time of its foundation in 1666 by *Louis XIV*, *Huygens* was a privileged member of the French Royal Academy of Sciences. Under unusual circumstances, he received an income of 6.000 French livers, granted to him after laborious negotiations, and he was accommodated at the Royal Library.

The French Royal Academy of Sciences was to become the arbiter of scientific thought. Its members were considered as educators who had the responsibility for the protection of knowledge and the evaluation of inventions, but were at the same time both judge and jury (2). *Huygens* participated actively in the activities of the French Royal Academy of Sciences and of note is his editorship of the research programs of the Academy. He was to resign from the Academy in 1681 and eventually become *persona non grata* in Paris after 1685 (Revocation of the Edict of Nantes).

2.3 - The Applications of the Cone in the 17^{TH} and 18^{th} Centuries

If the theoretical discussions on the glass cone can be found from the middle of the 17th Century, the practical application of a usable glass cone did not occur until a century later. That is due in part to the bad quality of glass available at the time. Furthermore, contrary to *Galilean* telescope glasses, the glass cone is subject to peripheral aberrations because of the smallness of the radius of curvature of the convex and concave components. To remedy this deficiency, present day magnifying systems, based on the glass cone principle, are composed of three elements of different index of refraction: a cone of barium glass of low refractive index for the central portion, and two lead or flint glass of high index for the objective and ocular.

According to *Moritz von Rohr* (3), the optician *Kirscher* had already proposed starting in 1646 the construction of usable "*Dutch telescopic spectacles*" as a magnifying system. In 1663, therefore, and before the posthumous publication of the work of *Huygens*, the British optician *Jacob Gregory* had already inspired the cone description in order to propose the construction of a telescope starting with a single lens as a visual aid for presbyopia. His son, *David Gregory*, had already taken up this idea in 1695 when he described the sphericity and thickness of the tube, the cone being put forward as a loupe. The studies of *von Rohr*, based on publications, catalogues and patents seems more exhaustive than those of *Levene* (4) who mentions that *Robert Smith* would have discussed any influence the length of the cone might have had on the magnification (1738).

^{2.} See the study on the French Royal Academy of Sciences by Brian & Demeulenaere-Douvere, 1996.

^{3.} Moritz von Rohr has published a large study on the history of Galilean telescopes and of telescopic cones and devices. (Rohr 1916a, 1916b, 1916c, 1918, 1920, 1925).
4. Levene 1977, p. 80.

3 - Huygens, the Neutralization of Corneal Diopter and Contact Lenses

In *Proposition XI* of his *Treatise on refraction and telescopes*, *Huygens* makes use of a terminology for his description of the contact cone, which could make one believe that he is defining a contact lens. He proposes, in effect:

"If you accustom to your eye a body made out of solid and transparent matter which also possesses a convex and a concave surface."

In the original Latin version: "Adaptetur ex solido materiae diaphanae frusto, cujus altera superficies convexa fit altera cava."

In the French version, "adaptetur" is translated by "si on adapte à l'oeil" (if one fits to the eye). In fact, the Latin version does not mention "the eye". In the context that we have described, it is evident that there is no contact by fitting the cone on the eye, but rather the placement in front and on the axis of the eye "of a solid and transparent body possessing a convex and a concave surface". This would be a good definition of a contact lens, if the textual sequel did not indicate "this body will enlarge distant objects", which does not allow of any ambiguity in regard to the intentions of Huygens: the "body" is certainly a magnifying cone and not an instrument for correction of a refractive error.

It is true:

- that *Huygens* has given a definition of a "body" which could lead to confusion with a contact lens,
- that he has drawn attention to the error of the *Descartes*' demonstration including speculation on lengthening of the eye to obtain enlargement of the image,
- that he has established in mathematical fashion the principle of the magnification system suggested by *Descartes*, which will later on be renamed "*Steinheil's cone*".

It is false:

- to be willing to attribute to *Huygens* any sort of priority of a contact system.

Nevertheless, *Huygens* has marked his epoch by his study of light and, in a more modest fashion, by his criticism of *Descartes*' system and this is the justification for the prominent position I attribute to him in this treatise.

APPENDIX

TRANSCRIPTION OF

Christian Huygens:

Tractatus de Refract. et Telesc. Liber II.1653 Propositio XI

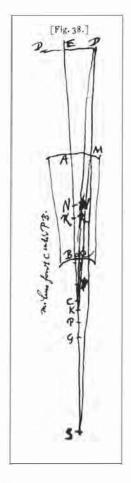
Theorema

Si loco conspicilli duarum lentium ejusmodi adaptetur ex solido materiae diaphanae frusto, cujus altera superficies convexa fit altera cava, eadem proportione visibilia augebit longinqua, atque conspicillum duarum lentium. Scilicit augmenti ratio ea erit, quae distantiae superficiei convexae a foco suo ad distantiam foci a cava, cui oculus admotus est.

Esto talis specilli continui superficies convexa AM [Fig. 38], ex sphaera cujus N centrum. Superficies vero BQ cava centro P. Et focus superficiei AM seu concursus parallelorum fit G punctum, at R punctum dispersus superficiei BQ radiorum parallelorum qui intra solidum feruntur. Porro visibile longinquum fit DED. Itaque ostendendum cum oculus superficiei B applicabitur ea proportione visibile DED augeri, quan habet AG ad GB.

Ponatur prius oculus in C non adhuc superficiei BQ prope admotus, et tribus hisce CR, CP, CB, ponatur quarta proportionalis CK, secundum prop. XII. Ergo quoniam adji ex C puncto si egrederentur, refracti in superf. BQ pertinerent ad punctum K, ideo vicissim qui intra diaphani soliditatem ita feruntur ut tendant ad K, pertinebunt post refractionem in superf. B ad punctum C. eadem ratione fi tribus hisce KG, KN, KA collocetur quarta proportionalis KS, fiet ut radij ad punctum S tendentes refractique in superficie AM tendant ad punctum K. Jungatur DS secant superf. AM in M. deinde MK secans superficiem BQ in Q, et connectatur QC. Recta vero DC secet superficiem BQ in O. Itaque radiorum ex puncto visibilis D is qui fertur secundum rectam DM, flectetur ab M versus K, sed iterum refractus in Q perveniet ad oculum in C. Quare constat in puncto Q superfitiei BQ spectari punctum D: quod spectaretur in O fi loco specilli, una tantum superficies B poneretur refractionis expers. Est igitur ratio magnitudinis apparentis ad veram oculo in C constituto, ea quae QB ad OB. Ration autem QB ad OB composita est ex rationibus QB ad MA; et MA ad ED; et ED ad OB, quae sunt eaedem rationibus KB ad KA; SA ad SE; et EC ad BC. Et est ratio composita ex rationibus SA ad SE, et EC ad BC, eadem compositae ex rationibus SA ad BC et EC ad SE. Itaque ratio QB ad OB componetur

ex rationibus KB ad KA, SA ad BC, et EC ad ES; ratio autem composita ex rat. KB ad KA et SA ad BC est eadem compositae ex rat. KB ad BC et SA ad KA, reliqua vero EC ad ES est ratio aequalitatis, quoniam visibile DED longinquum ponitur. Ergo ration QB ad OB composita ex ratione KB ad BC et SA ad KA. Quia vero ex constr. est CR ad CP ut CB ad CK, erit PR ad RC ut KB ad BC. Item quia KG ad KN ut KA ad KS erit NG ad GK ut SA ad AK. Igitur ratio QB ad OB componetur ex rat. PR ad RC et NG ad GK, oculo adhuc in C constituto. Cum vero superficiei BQ oculus contiguus ponetur cadet C in B, item K in B, quare tunc erit ration PR ad RC feu RB eadem I quae est refractionis, ac proinde eadem rationi AG ad NG. Ratio vero NG ad GK erit NG ad GB. Ergo tunc ration QB



ad OB, quae est ratio magnitudinis apparentis ad veram erit composita ex rat. AG ad NG et NG ad GB hoe est, erit ea quae AG ad GB; quo erat demonstr.

Oportet autem superficiem BQ certa ratione cavam esse si distincta visio requiritur. Nam alioque etsi magis minusve cava esset, aut plana aut convexa quoque, idem prorsus contingeret augmentum, si modo oculus prope admotus ponatur. Nam semper eadem demonstratione ostendetur magnitudinis apparentis ad veram esse rationem eandem, quae AG ad GB.

Hisce vero nequaquam consentiunt ea quibus Cartesius Telescopij inventum explicare contendit, similem huic tubum proponens solidum. Vult enim cavam superficiem ejusmodi esse ut radios a singulis visibilium punctis procedentes et per superficiem tubi exteriorem transmissos, ita inflectat ac ad oculum mittat tanquam si a propioribus punctis advenirent. Et quam rationem habuerit distantia horum punctorum propinquiorum ad distantiam ipsius visibilis, eandem magnitudinis apparentis ad eam quae folis oculis perciperetur definit. Hoc autem quomodo verum fit, quum senum oculis ea conveniat telescopij constitutio, ut radij convergentes aut certe paralleli ad oculum deferantur, non autem quasi ex puncto aliquo propiori manantes. Et notum est tamen non minus senibus quam qui visu pollent specierum magnitudines multiplicari.

Porro illud quoque in eadem Cartesij explicatione absurdum, quod eam ob causam majora omnia videri ait, quoniam ex diversis rei visae punctis venientes radij decussentur in exteriori convexa tubi superficie, qui tubo non adhibito ad pupillam oculi decussarentur; quoniam enim si plana aut concava esset loco convexae superficiei nihilominus decussatio similis ibi contingeret efficietur aeque etiam inverso tubo majora omnia conspici debere. Quod ijs quae superius demonstrata fuere atque ipsi adeo experientiae adversatur.