Hist Ophthal Intern 2018, Vol.2: 127-151

The Experimental Study of Binocular Vision by Ibn al-Haytham and Its Legacy in the West*

Dominique Raynaud**

Abstract
Early modern physiological optics introduced the concept of correspondence through the study of the conditions for the fusion of binocular images. The formulation of this concept has traditionally been ascribed to Christiaan Huygens (1667) and to an experiment often attributed to Christoph Scheiner (1619). However, Scheiner's experiment had already been conceptualized, first by Ptolemy (90–168 AD), then by Ibn al-Haytham (d. after 1040). The extent of the latter's knowledge of the mechanisms of binocular vision is analyzed. It is explained why Ibn al-Haytham, who addressed this problem as an experimentalist, succeeded in discovering the theoretical horopter (the locus of points in space that yields single vision) and yet failed to recognize that the horizontal line of the horopter could be described as a circular plane around the viewer's head, credit for which goes to Vieth (1818) and Müller (1826). Through his experiments, Ibn al-Haytham established the notion of corresponding points, explored the cases of homonymous (direct) and heteronymous (crossed) diplopias, and prepared the ground for the discovery of Panum's fusional area. Finally, the influence of al-Haytham's pioneering treatise *Kitāb al-manāẓir (Book of Optics)* on Western science is examined, by studying the translations and commentaries that were available in the Latin world.

Introduction

A key chapter in the physiology of optics considers the conditions for the fusion of the quasi- or displaced images generated by the two eyes. Interestingly, the ancient Greeks did not explore the questions raised by binocular vision in any depth. Euclid only devotes three propositions to this problem (*Optica*, prop. 26–28) [33,80] and limits his analysis to what is seen of a sphere in binocular vision. If the diameter of the sphere is less than, equal to, or greater than the inter-pupillary distance, then the two eyes will perceive a spherical cap that is greater than, equal to, or less than the hemisphere, respectively. Nowhere does Euclid address the question as to how disparate visual stimuli are integrated.

Galen attempted to rectify this lacuna by deriving a definition of the disparity between the quasi-images produced by the two eyes from one of Euclid's propositions (*Optica*, prop. 30). Unfortunately, Galen's exposition on binocular vision is extremely brief [21] and its primary aim is to provide an anatomical description of the ocular paths.

Ptolemy's treatment of binocular vision is far more comprehensive (*Optica*, II, 27–46, III, 25–62), [39A,10] providing the framework for Ibn al-Haytham's research on the same subject. A comparison of the texts by the two savants confirms that many of the results obtained by Ibn al-Haytham were based on the thorough study and critical analysis of his predecessor's work. Nevertheless, it would appear to be more useful to take the work of Ibn al-Haytham rather than Ptolemy as our departure point to study the development of the binocular theory of vision. This choice can be justified on two grounds.

Firstly, while Ptolemy may have furnished the original matrix for Ibn al-Haytham's theory, his ideas are presented in *Optica* in anything but a systematic manner, with the

**Author´s address:
Dominique Raynaud
Université Grenoble Alpes
UFR SHS, CS 40700
F-38058 Grenoble Cedex 9
France
eMail: dominique.raynaud@univ-grenoble-alpes.fr

*The text is adapted from D. Raynaud, *Studies on Binocular Vision*, Springer, 2016: 71–114. DOI 10.1007/978-3-319-42721-8_5.
Permission granted by Springer.com

propositions regarding binocular vision straddling Books II and III. What is more, certain results appear to be quite tentative, a fact that earned him the criticism of Ibn al-Haytham: "[Ptolemy] said that when the eye gazes at the middle object assumed in the middle of the ruler at the point intersection of the two diameters, then the two lines or diameters representing the visual axes will be seen as a single line that coincides with the common axis… But it is an error attested both by reasoning and experience." [30] In the final analysis therefore, Ptolemy can contribute little to our understanding of the history of binocular vision, because at each turn we find ourselves having to explain elements that require modification or amendment.

Secondly, unlike his work *Almagest*, which circulated widely and was universally read between the Duecento and the Cinquecento, Ptolemy's treatise on optics never achieved the stature of an authoritative text in Europe. In contrast, it appears to have been better known in the Arab world; we find it cited two times in Greek and five times in Arabic compared to just twice in Latin treatises from the Middle Ages. Historians have suggested that the eclipse of Ptolemy's *Optics* can be explained by the appearance of Ibn al-Haytham's *De aspectibus*, which was read by the Latin perspectivists and would orient their work. As Smith wrote: "With the continued dissemination of Ibn al-Haytham's treatise and the proliferation of Perspectivist works during the late thirteenth and early fourteenth centuries, Ptolemy's *Optics* was bound to lose its status as a legitimate source in optics." [76] In fact, another reason not to take Ptolemy as our reference point is because Latin authors retained that the explanation of binocular vision was not to be found in Ptolemy's *Optics* but in Ibn al-Haytham's *De aspectibus*.

It is therefore to the Arab world that we must turn in our search for the key to a correct reading of the further development of the theory of binocular vision. As we begin to re-trace this history, one of the first texts that deserves mention is the *Book of the Causes of the Diversity of Perspectives in Mirrors* by the Melkite physician and scientist Qusṭā Ibn Lūqā (820–912), who settled in Baghdad and translated many ancient Greek texts into Arabic. Qusṭā Ibn Lūqā draws the distinction between pathological diplopia and physiological diplopia and poses the question:

> By what cause is one single thing seen twice or more? And in how many ways is this possible? We have said earlier that the sense of sight perceives visible [things] if the visual ray falls upon them. The visible will be seen as unique if only one ray falls upon it; if two visual rays fall on it, it will be seen as double … If it happens that the two cones emerging from the two eyes separate from each other, such that a radiant cone from each of the two eyes falls on the same visible [thing], then the same thing will be seen as double. [60A]

He then constructs a classification of the causes – both natural and artificial – which could result in the separation of the visual cones and cause diplopia; among these are strabismus and the image of two objects located at different distances:

> It may happen that one single thing will be seen as double … when a man fixes his pupil on a close thing and on another thing in the direction of that [thing] which he has fixed on but which is further than it from the eye; it is then that he will see one single thing as two. In effect, when he looks at the closer [thing] and fixes on it, one of the two rays [from the two eyes] will bend in relation to the other, and the two rays will fall on the more distant visible [object] in this manner; as a consequence he will see two things. [60B]

The term "artificial cause" used by Qusṭā Ibn Lūqā, and the fact that he classifies among these causes both pathological conditions (such as strabismus) and accidental circumstances (e.g., pressure exerted on the eyeball) could give rise to confusion. However, he clearly states that diplopia may occur under normal as well as abnormal conditions. The disparity arising from the observation of objects situated at different distances does not necessarily imply that there is a problem with the visual apparatus; it is *normal physiological diplopia* and one cannot apply the theory of neutralization here to explain the integration of two sets of visual sensations.

Ibn al-Haytham (d. after 1040) was the second Arab savant to turn his attention to binocular vision. He is unanimously recognized for his work in three fields: mathematics (*Commentary on Euclid's Premises; On the Completion of Conics; Exhaustive Treatise on the Figures of Lunes; On the Regular Heptagon; On the Measurement of the Paraboloid;* and *On the Measurement of the Sphere*, a short tract on what came to be known in number theory as Wilson's theorem, although al-Haytham was the first to state it), astronomy (*On the Determination of the Meridian from One Solar Altitude; On the Visibility of Stars; Doubts on Ptolemy; The Resolution of Doubts on the First Book of the Almagest; The Resolution of Doubts on the*

Winding Movement), and optics (*On Optics; The Discourse on Light; On Spheric Burning Mirrors; On Parabolic Burning Mirrors; On the Burning Sphere; On the Light of the Moon; On the Halo and the Rainbow; On the Formation of Shadows; On the Shape of the Eclipse;* etc.). His studies on binocular vision are less well known.

Ibn al-Haytham drew inspiration from Ptolemy, [30B] – we are certain of the existence of this link based on a passage in Doubts on Ptolemy, in which Ibn al-Haytham explicitly mentions the experiments of his predecessor [30, 66] – Ḥunayn Ibn Isḥāq [27, 49] and perhaps also Qusṭā Ibn Lūqā, [60A] building on their work but approaching the same questions with greater rigor. Ptolemy provides only a brief description of the instrument devised by him to study binocular vision. In contrast, Ibn al-Haytham's text allows us to reproduce the experiments and understand his results without referring to other sources. [30, 69A] Ibn al-Haytham's corrections to his Greek predecessor's work were passed on to successive generations, confirming that the medieval Latin authors followed his lead rather than Ptolemy's. For example, he set up and conducted an *experimental study* on the conditions for the fusion of the quasi-images generated by two eyes, as we will describe below, producing an impressive series of results. Not only did he identify the existence of corresponding points on the recipient surfaces of the two eyes and differentiate between the cases of homonymous (direct) and heteronymous (crossed) diplopia; he also conceived a credible model for the horizontal horopter and anticipated the discovery of Panum's fusional area, [62, 26] despite the fact that the medieval world was not yet in possession of the Keplerian concept of the "retinal image". [32] It should be recalled that the retinal image is not sharp on all parts. As with any centered optical system, the eye is not subject to the conditions of strict stigmatism. The deviation from the paraxial approximation, which is written as a sum of six factors, characterizes the various types of geometric aberration (spherical aberration, astigmatism, coma, field curvature and distortion). The two main factors that distort the image are spherical aberration and astigmatism. It suffices to fix one's gaze ahead to realize that an object seen through peripheral vision is indistinct; therefore the retinal image only meets the conditions of approximate stigmatism.

The experiments conducted by Ibn al-Haytham on binocular vision are presented in his seven-volume treatise on optics, *Kitāb al-manāẓir* (III, 2). Chapter 2 is divided into six sections. In paragraphs 2.1–2.24 the author defines various terms and concepts and the qualitative approach that he would adopt in analyzing the problem of binocular vision. Paragraphs 2.25–2.50 present the results of a series of experiments conducted using a board that he referred to as a 'binocular ruler' on which small columns of wax could be positioned. Paragraphs 2.51–2.54 compare the binocular and monocular perception of diagonal lines drawn on this tablet. Paragraphs 2.55–2.61 describe similar experiments, in which the columns of wax are replaced by words written on a piece of parchment. Paragraphs 2.62–2.85 explore the visual perception of objects whose direction is gradually displaced further and further from the anterior-posterior axis. Paragraph 2.86 presents Ibn al-Haytham's conclusions, and announces the analysis of "visual illusions" carried out in chapter 3. Ibn al-Haytham employed the term *aghlāṭ al-baṣar* / *deceptiones uisus*, which has the same root as 'error' *(ghaliṭa: errare / decipi)*, but sometimes used the word *īhām (illusio)* instead. He was not the only scientist to consider (physiological) diplopia as normal. Bacon also did so; in his exposition of errors in vision, he mentioned only strabismus, the effects of cold or warm temperatures, passion, nervous derangement, problems with the vitreous humor, compression of the eyeball, obstruction of the lens, and the double pupil. [6A] What occurs when one fixes one's gaze on objects situated at different distances is discussed elsewhere by Bacon. His study of physiological diplopia is entitled "In quo ostenditur duobus diuersis experimentis et diuersis figurationibus, quomodo unum uideatur duo" (*Perspectiva*, II, II, 2). John Pecham no longer treated diplopia in the context of errors in vision (*Perspectiva communis*, I, 80). [52A]

The cases of homonymous and heteronymous diplopia

There are two types of diplopia. The first is referred to as heteronymous or crossed diplopia, and the second as homonymous or uncrossed diplopia. The following experiment is often used to illustrate how the two forms of physiological diplopia manifest themselves:

> The phenomenon of physiological diplopia can be made to happen by means of an experiment devised long ago by Scheiner. That is to say, [take] a small wooden ruler about 50 cm in length, one end of which is placed at the tip of one's nose, and which has been pierced by

> three pins at 30, 40 and 50 cm. If one gazes fixedly at the middle pin, the two other pins will be seen as doubled.[70A]
> When the ruler is pricked with a black-headed pin and a white-headed pin and one fixes one's gaze on the black pinhead: a) if the white pinhead is located closer to the eyes than the black pinhead … there is crossed diplopia; b) if the white pinhead is located further from the eyes than the black pinhead … there is direct diplopia. [78]

This demonstration has traditionally been attributed to the astronomer and mathematician Christoph Scheiner (1619), but it actually goes back much further [71] because both Ptolemy and Ibn al-Haytham devised similar experiments and came to the same conclusions. This ignorance of the historical facts brings up the issue of tacit borrowings. How was it that a seventeenth-century Jesuit came to be credited with an experiment carried out long before by a scientist in ancient Alexandria and another in medieval Cairo? Ptolemy's work on binocular vision has been analyzed in depth by Lejeune, [38] so we will focus on the research of the Arab savant.

Ibn al-Haytham begins by describing the instrument that he designed for the study of binocular vision. It consists of a flat, rectangular piece of wood *ABCD* (*lawḥ, tabula*), one cubit in length (45 to 50 cm) and four fingers wide (6 to 7 cm.). At one end (*AB*) is a shallow depression (*MHN*) which serves as a nose rest. Lines are drawn across the ruler (in different colors to make them more visible); the median lines *HZ* and *KT* (*khaṭṭān, linee recte*) bisect the length and width of the ruler, and the diagonals *AD* and *BC* cross it (*quṭrān, diametra*). [31A] Ibn al-Haytham then fashions three small columns of wax painted in different colors and positions them at various points along these lines. With this instrument he conducts a series of experiments on the conditions for the fusion of binocular images that would serve as a model for the study of optics in the West for centuries.

The notion of corresponding points

Modern specialists in physiological optics date the introduction of the notion of "corresponding points" as an explanation for binocular vision to the seventeenth century and the research of Christiaan Huygens (1629–1695). [28, 29] As Pigassou-Albouy observed:

> The earliest notion of corresponding retinal points can be traced back to Huygens: "[…] every point at the back of the eye has a corresponding point at the back of the other, such that when one point of an object is painted in several pairs of these corresponding points, then it will appear simply as it is." [54]
> To explain the unification of the two visual sensations, Huygens and Müller denominated 'corresponding retinal points' (CRP) the photoreceptors whose stimulus simultaneously produces the sensation of a single source. [55, 37A]

The concept of "correspondence," today defined as the association between the nasal point of one retina and a temporal point in the other retina (see below *Figure 2*, points p_L and p_R) considerably antedates the work of Huygens. One already finds the adjective *consimilis* applied by Ptolemy to visual rays. These were the "similarly arranged rays" (*radii ordine consimiles*) translated by Lejeune as "corresponding rays," [24C] which accords with the modern usage of the term but was only codified during the nineteenth century. As Helmholtz wrote:

> We will assign to them the terms *coincident* or *corresponding* points; they have also been referred to as *identical* points, in keeping with a particular theory. Since to each point in every visual field there corresponds a specific retinal point, one may refer to these interchangeably as *coincident, correspondent or identical points in the two retinas*. [24, 79, 81]

These historical differences in terminology, which are echoed in Helmholtz's text on corresponding retinal points, are simply the result of the difficulty of finding an exact translation for the Latin word *consimilis*. Ptolemy limits his application of this adjective to visual rays, but Ibn al-Haytham uses it explicitly in his discussion of corresponding points:

> So the two forms (*al-ṣūratayn*) impressed on the two points that are correspondingly situated *(fī nuqṭatayni mutashābihatay al-waḍʿ, in duobus punctis… consimilis positionis)* with respect to the surfaces of the two eyes reach the same point in the hollow of the common nerve, and they will be superimposed on that point so as to produce a single form. [31B]

Here *ṣūra* (form) designates the sensory image rather than the image produced by the eye, which Ibn al-Haytham denominated *al-khayāl*. The "surface of the eyes" (*saṭḥ al-baṣar, superficies uisus*) referred to here is the anterior surface of the crystalline lens (*al-jalīdiyya, anterior glacialis*), which was thought to be the seat of sensory responsiveness and which Ibn al-Haytham termed "the surface of the sensory body" (*saṭḥ al-jism al-ḥāss, superficies uisus sentientis*). [31C, 68]

This is not the retinal surface, which func-

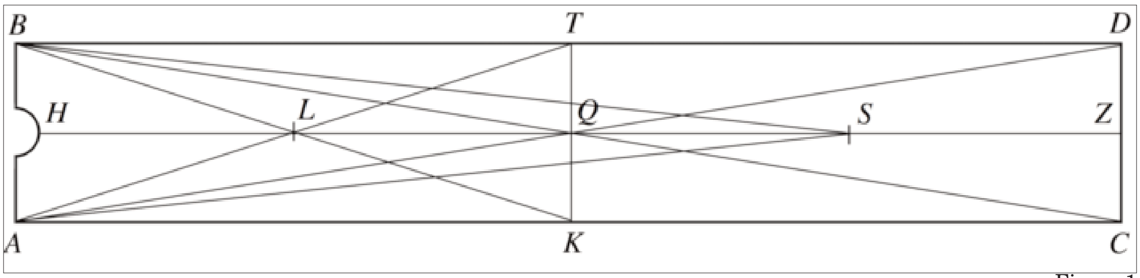

Figure 1
Ibn al-Haytham, Experiment 1.

Author's drawing.

tion was first understood by Platter [56] and Kepler, who wrote in *De modo visionis*: "I say that vision occurs when the image of the whole hemisphere of the world that is in front of the eye, and a little more, is formed on the reddish white concave surface of the retina." [32] The first description of the role of the crystalline lens in projecting visual images onto the retina would be provided by Scheiner. [71] Moreover, Ibn al-Haytham lists the parts of the eye (*ruṭūba al-bayḍiyya*: humor albugineus, *r. al-jalīdiyya*: crystallinus, *r. al-zujājiyya*: humor vitreus, *ṭabaqa al-ʿankabūtiyya*: tela aranea, *ṭ. al-multaḥima*: consolidativa, *ṭ. al-ʿinabiyya*: uvea, *ṭ. al- qarniyya*: cornea), but says nothing of the retina. We may ask ourselves whether the rectification of this lacuna would have been significant; in the Middle Ages the most authoritative text on the anatomy of the eye was *The Book on the Ten Treatises on the Eye*, whose author, Ḥunayn Ibn Isḥāq, limits the role of the retina (*ṭ. al-shabakiyya*) to that of providing nutrients to the vitreous humor and the crystalline lens. [46, 69B, 64, 65] Thus, the role of the retina constitutes the principal point of divergence between medieval and early modern theories of physiological optics.

Setting aside this difference, Ibn al-Haytham deserves credit for having formulated the concept of correspondence, which would serve as the starting point for the analysis of many aspects of binocular vision. It is possible that Huygens developed his own theory of corresponding points after reading Ibn al-Haytham's treatise, which was well known in classical Europe through the edition published by Risner in 1572 *(Opticae thesaurus: Alhazeni Arabis libri septem).*

The study of physiological diplopia (Experiments 1 and 2)

According to early modern physiological optics, the fusion of two images into one in binocular vision takes place when the object points are 'painted' in corresponding points on both retinas. The phenomenon of *diplopia* arises in cases where the object points fall on disparate points on the two retinal surfaces. In *homonymous diplopia* every object situated *on the further side of* the point of fixation is seen as a double image, whereas in *heteronymous diplopia* every object situated *on this side of* the point of fixation is seen as a double image. Neither Ptolemy nor Ibn al-Haytham employed a scientific term when referring to diplopia (using instead *comprehendetur duo* and *yudraku 'ithnayni*), but they nonetheless make a clear distinction between the two types of physiological diplopia. For example, Ptolemy writes: "If line *HTK* is drawn parallel to line *EDZ* while the two axes remain focused on *D*, an object at point *T* [which is located below the point of fixation *D*] will appear at the two locations *H* and *K* ... But if we focus both axes upon point *T*, we will see point *D* [which is now located beyond the point of fixation *T*] at points *E* and *Z* / Cum autem producta fuerit linea *HTK* equidistans linee *EDZ* et fuerint duo axes oppositi puncto *D*, res que est super punctum *T*, videbitur in duobus locis qui sunt *H*, *K*... Et si posuerimus utrosque axes oppositos puncto *T*, videbimus tunc *D* super punctos *E*, *Z*," [39D, 38A] Here we describe the experiments conducted by Ibn al-Haytham using his binocular ruler.

Experiment 1. [31D] When columns of wax are situated along the same diagonal at points *L Q S* with the eyes fixed on *Q*, double images of the columns at *L* and *S* are seen (*Figure 1*). As has already been said, the doubling of the column positioned at *S*, beyond the point of fixation, illustrates the phenomenon of homonymous diplopia, while the doubling of the column at *L* illustrates crossed diplopia.

Experiment 2. [31D] When columns of wax are situated along the same diagonal at points *I Q P* and the eyes are fixed on *Q*, double images of columns *I* and *P* are seen, and likewise if the columns are situated on this

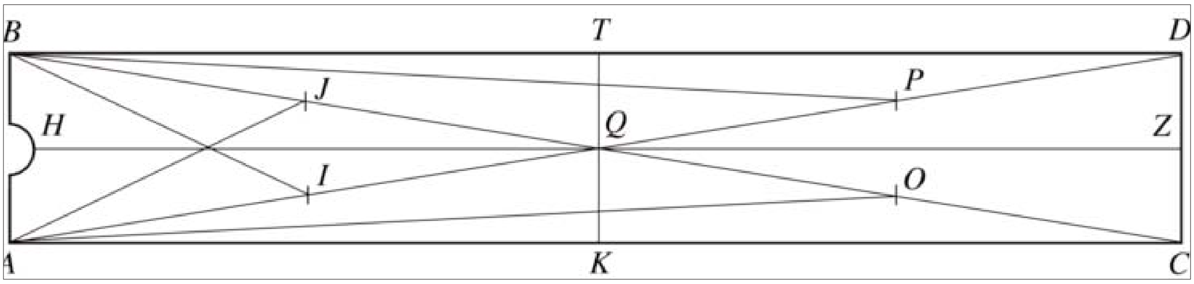

Figure 2
Ibn al-Haytham, Experiment 2.

Author's drawing.

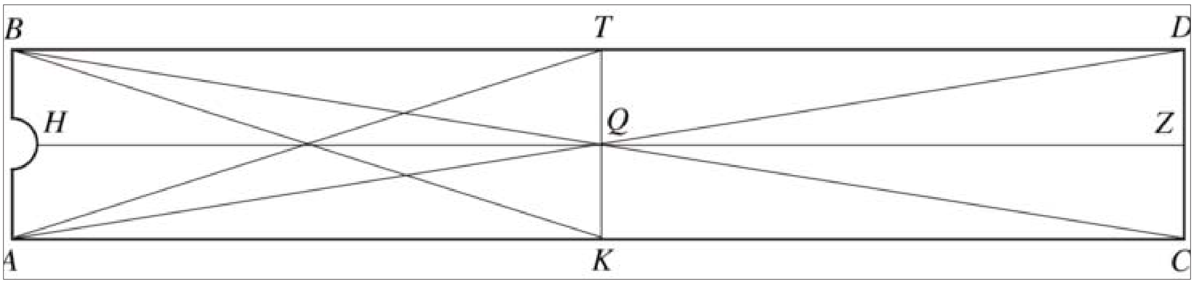

Figure 3
Ibn al-Haytham, Experiments 3 and 4.

Author's drawing.

side of the fixation point, on $I\ J$, or beyond the point of fixation, on $O\ P$ (*Figure 2*).

The determination of the horopter (Experiments 3 and 4)

The concept of the *horopter* was introduced in 1613 by the Jesuit mathematician and physicist Franciscus Aguilonius. [2] He defined the horopter as the frontal plane containing the point of fixation. Following Aguilonius, and principally in the nineteenth century, the horopter formed the object of many studies, notably those of Vieth, [82] Müller, [44] Hering [25] and Helmholtz. [24] Today the horopter is limited to the horizontal plane and is defined by the locus of points in space received by corresponding points on the two retinas. Although both the name and the form of this geometric locus based on the notion of corresponding points were unknown to Ibn al-Haytham, his work led directly to its discovery.

Experiment 3. [31D] When the three columns of colored wax are situated at $T\ Q\ K$ with the eyes $A\ B$ fixed on Q, single images of the columns are seen (*Figure 3*).

Experiment 4. [31D] The same occurs if one takes T or K as the fixation point without displacing the wax columns (*Figure 3*).

This represents an early attempt to determine the horopter scientifically, which according to Ibn al-Haytham corresponds to the frontal line $T\ K$ (Aguilonius would come to the same conclusion in the seventeenth century). The historian Abdelhamid I. Sabra recognized clearly that Ptolemy and Ibn al-Haytham laid the groundwork for the discovery of the modern horopter, through which object points are re-composed as single images. He wrote:

> It would have been easy for Ptolemy and Ibn al-Haytham to generalize this conclusion further. For points in the plane of the axes, the stated conditions of single vision (taken literally) are satisfied only by points on the circumference of the circle passing through the centers of the eyes and the point of fixation (the so-called 'horopter circle' or 'horizontal horopter')… But neither Ptolemy nor Ibn al-Haytham draws this consequence. (Note, however, that Ibn al-Haytham's account is not strictly geometrical). [69C]

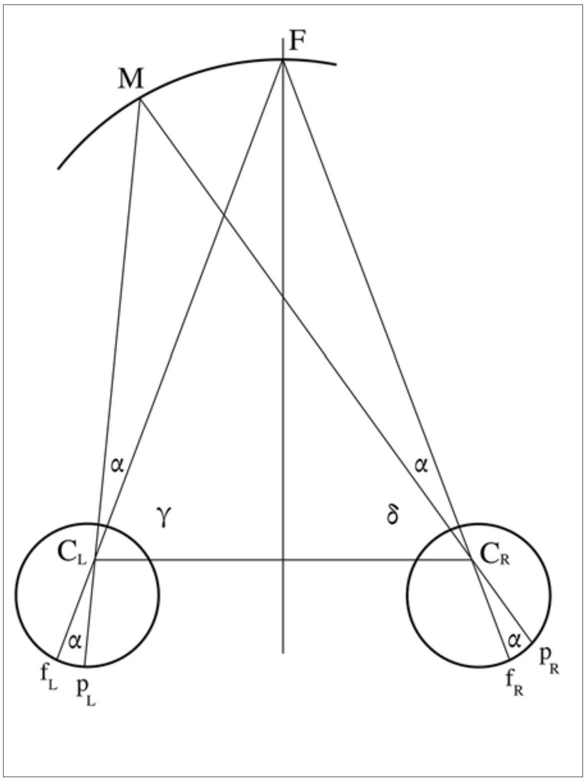

Figure 4
Vieth-Müller horopteric circle.

Author's drawing.

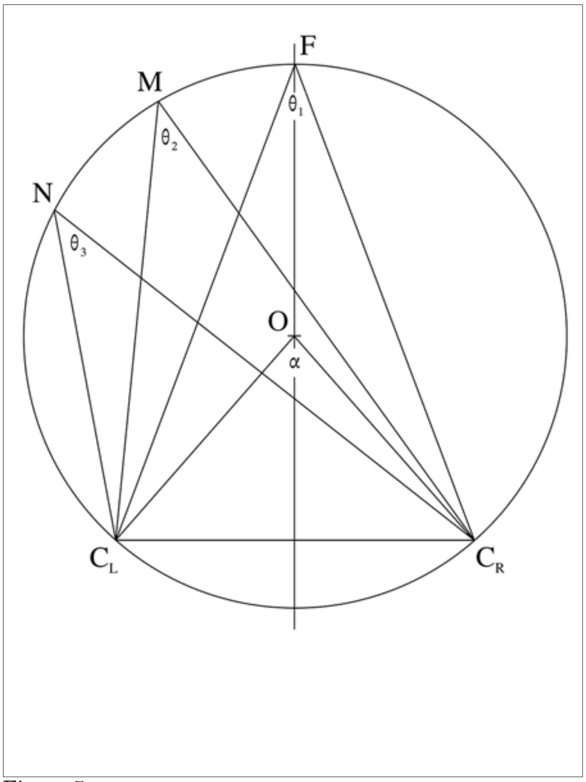

Figure 5
The theorem of inscribed angles.

Author's drawing.

This simple observation invites us to reexamine Ibn al-Haytham's experiments in order to determine why he did not arrive at the true form of the horopter.

The theoretical horopter
First of all, let us retrace the genesis of the Vieth-Müller horopteric circle. We will limit ourselves to a consideration of the horizontal plane of vision that allows us to define the "longitudinal horopter". What is the geometric locus of the object points that are seen as a single image by the two eyes?

Let C_L be the optic center of the left eye, C_R the optic center of the right eye, and F the fixation point. Let γ designate the angle FC_LC_R and δ the angle FC_RC_L. As they fix on point F, the rotation of the two eyes will produce quasi-images of F on the foveae f_L and f_R (*Figure 4*). The object point M will be received in the retinas of the two eyes by the corresponding points p_L and p_R, that is to say, by the corresponding nasal and temporal sensory cells in the two eyes.

Under these conditions the visual directions p_LM and p_RM will correspond as well, and form equal angles (α) as they cross the arcs f_Lp_L and f_Rp_R. M then becomes the point of intersection of the lines with an angle of $\gamma + \alpha$ and $\delta - \alpha$ in relation to the line connecting the optic centers. One may immediately deduce from this

$$\begin{aligned} C_LFC_R &= \pi - \gamma - \delta \\ &= \pi - \gamma - \alpha - \delta + \alpha \\ &= C_LMC_R \end{aligned}$$

The theorem of inscribed angles teaches us that on a circle with a center O and two base points C_L and C_R, a point M describing the circle will determine an inscribed angle C_LMC_R that is constant and equivalent to one half of the center angle C_LOC_R.

In the present case (*Figure 5*), the object points that are seen as single images by the two eyes are points F, M, N … such that

$$\theta_1 = \theta_2 = \theta_3 = \ldots = \frac{\alpha}{2}$$

These are the grounds on which Vieth and Müller defined the theoretical horopter as the circle passing through the two optic centers C_L and C_R and the fixation point F. A given fixation point will be associated with a unique horopteric circle. Let us now suppose that F

is displaced along the line *OF*. The horopter will then be modified and, as we continue to move *F*, all of the horopters will form a linear pencil of circles with C_L and C_R as their base points (these points will have a zero power in relation to all the circles of the pencil).

We now arrive at a question that is of great interest to the history of optics: having come this far, why did Ibn al-Haytham fail to construct the horopteric circle? A series of considerations may be raised in this regard.

1) Did the Arab scholar have the mathematical knowledge necessary to realize such a construction? Yes. The property of the geometric plane on which the horopter is based had been known since antiquity: it was expounded by Euclid (*Elements* III, 20–21), [18] Archimedes (*Book of Lemmas*), [3] and al-Kindī (*Rectification of Errors*). [60C] Ibn al-Haytham, who wrote a treatise entitled *Solution of the Difficulties and Explanation of the Notions of Euclid's Book (Kitāb fī Ḥall shukūk kitāb Uqlīdis wa sharḥ maʿānīh)*, was clearly aware of the theorem of the inscribed angle.

2) Did Ibn al-Haytham possess the concepts of physiological optics necessary to trace the horopter? Müller (1826) drew upon Huygens's notion of correspondence in his construction of the horopteric circle, but there is no reason to date the emergence of the concept to the seventeenth century because, as can be demonstrated, Ibn al-Haytham was already using it in the early eleventh century (*fī nuqṭatayni mutashābihatay al-waḍʿ, in duobus punctis consimilis*). Müller's only genuine innovation was to apply the concept of correspondence to points in the *retina*, but this insight was in no way essential to the model of the horopteric circle. The path of a ray entering the eye being determined by the indices of refraction of the different media that it is crossing, the correspondence applies to *the entire length of the optic path*. One is therefore justified in using the term correspondence with regard to points in the retina, the crystalline lens, the cornea, and other parts of the eye along the optic path. Ibn al-Haytham, who placed the site of sensibility in the crystalline lens, could have constructed the horopter beginning with the corresponding points in the lens.

3) Did Ibn al-Haytham rely on an authoritative argument in constructing the horopter? He would have found, for example, in Ptolemy: "So too, [with the two eyes located at points *A* and *B*, and the visual axes fixed to *D*], if we draw a line *EDZ* through point *D* perpendicular to *GD*, any object place on this line will appear single and at its true location as long as it is aligned with point *D*" [39F] (*Figure 6*).

This proposition is almost equivalent to the one advanced by Ibn al-Haytham, but there is no reason to believe that he was merely repeating an authoritative argument. As has been seen, in *Doubts on Ptolemy* Ibn al-Haytham rejected certain conclusions arrived at – supposedly on the basis of experimental evidence – by his predecessor. Since Ibn al-Haytham made such a critique of Ptolemy, it is difficult to admit that he would have borrowed another of the latter's propositions without having tested it.

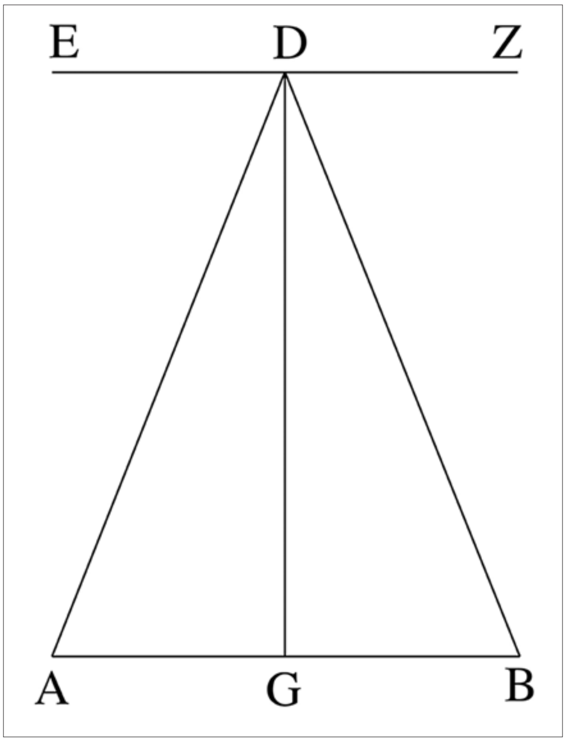

Figure 6
Ptolemy, Optics III, 26.

Author's drawing after
Lejeune, *L'Optique de Claude Ptolémée*, p. 103.

4) There is, all the same, another reason that could explain why Ibn al-Haytham was not able to arrive at the demonstration of the horopteric circle: he approached the problem as an experimentalist rather than a geometer. [46B, 58, 59, 72, 67, 74] If there was a marked gap between the theoretical and the experimental horopter, there would no longer be any reason to judge the conclusions of Ptolemy and Ibn al-Haytham as unsatisfactory in comparison to the horopteric circle of Vieth-Müller, the latter being the narrow interpretation

adopted by Albert Lejeune and Abdelhamid I. Sabra. [39F, 69C] And yet modern ocular physiology continues to distinguish between the two models. Helmholtz repeated Hering's experiments but was unconvinced by the results. He wrote: "As for me, when I place myself at the distance indicated by Hering, the surface of the threads seems to me to be distinctly concave [whereas they should appear flat]". [24B] And one finds in the literature:

> On a horizontal plane, the horopter corresponds *roughly* to the circle determined by the nodal points of the two eyes and the fixation point. [78B]
> Different forms of the horopter have been proposed … The empirical horopter of the horizontal correspondences appears to be more valid than the geometric horopter. [55B]

The distinction that is regularly made between the 'theoretical horopter' and the 'empirical horopter' reflects the doubts that contemporary physiological optics continues to raise with regard to the geometric model of Vieth-Müller.

The experimental horopter

It appears in fact that the geometrical construction of the theoretical horopter just described is based on some significant simplifications. [36]

1) The visual field in which the horopter can actually be tested is limited. First of all, the binocular field of vision is framed by the temporal monocular fields of the two eyes; in a normal subject the limiting nasal ray forms an angle $\alpha = 55°$ with respect to the geometric axis of straight vision. Given a fixation point located at a distance of 25 cm (as in Ibn al-Haytham's experiment), the two eyes can only perceive objects located in a sector of $2\alpha = 110°$, which represents a center angle of 190° on the horopter.

One must also take into account Mariotte's blind spot, the physiological scotoma discovered by Edme Mariotte in 1667, which results in two zones of monocular vision within the full binocular field.

Finally, the iris does not lie in the median plane of the eye, but approximately 3.8 mm in front of the optic center. And since the iris has an average diameter of 3.5 mm, rays that penetrate the eye at an angle of less than 27° in relation to the direction of the fixation point will pass through the nodal points. If we designate the nodal point object as N, the internal limits of the iris as I and I', and the center of the segment II' as H, then angle 2β below which a visual ray can touch the nodal point is determined by the relationship:

$$\beta = \arcsin\left(\frac{HI}{HN}\right)$$

Since $HI = 1.75$ mm and $HN = 3.8$ mm, then $\beta = 27° 25'$.

The more oblique incident rays will either be masked by the iris or refracted to the periphery of the crystalline lens. Their path will then be subject to spherical aberration. Under the same conditions, vision will be distinct for the points contained within a sector of 69° (cutting a center angle of 108° on the horopter).

2) If one attempts to correct for this situation by inducing mydriasis (increasing the diameter of the pupil to its maximum of 8 mm) the oblique incident rays will pass through the nodal points, but because the pupil is dilated most of the rays will be refracted to the periphery of the crystalline lens. In fact, the larger the diameter of the pupil, the greater the confusion created by spherical aberration.

3) Physiological optics utilizes a geometric model of the eye in which the retinal surface is likened to the portion of a sphere. But as Yves Le Grand acknowledged, "This geometric schema is a fiction. In reality … the geometric axis [the anterior-posterior zz'] is not an axis of rotation; in the equatorial plane, the curvature of the ray is generally smaller on the temporal side than on the nasal side." [37B] Thus, when a point issuing from the horopter reaches the retinas it will not strike two exactly corresponding points, because the corresponding rays will reach different retinal cells.

4) Another difficulty lies in the fact that Listing's law [42] – which states that the ocular globes are subjected to a torsion that increases with the distance of the fixation point from the horizontal or vertical axis – only holds in the case of a line of sight focusing on infinity. When the eyes move from the primary position and converge on a fixation point that is quite close, a simple movement of intorsion around the axis yy' should be produced.

> And yet this does not take place; when the fixation point is nearer, the meridians are displaced upward, which means that the convergence leads to the outward rotation of

the two eyes … The displacements in the horizontal gaze predicted by Listing's law reach 1 degree for every 10 degrees of convergence in the angle 2β of the lines of sight. [36A, 70B] [Taking as his departure point a discovery by Volkmann, Helmholtz had already demonstrated that, as the fixation point is moved closer to the eye, a torsion around the anterior-posterior axis occurs, such that "the convergence leads to deviations of 2° to 2½° in the accidental image"]. [24C]

For the left eye, the image of a horizontal segment will pivot in an anti-clockwise direction; for the right eye, the image will pivot in a clockwise direction. The distance separating the two stimulated visual cones is 4.5 μm; therefore images form on corresponding retinal points only if the torsion around the axis zz' is less than $1'$ of the arc. For a fixation point situated 25 cm from the eyes (as in Ibn al-Haytham's experiment) the combined torsion of the eyeballs will be $2\alpha = 3° \; 8''$. The angle of convergence 2β corresponds to the angle $C_L F C_F$ formed at the fixation point F by the visual axes of the eyes C_L et C_R. If H is the point of intersection between the median axis and the line connecting the optic centers $C_L C_R$, one has:
For $C_L C_R$ = 6.5 cm and FH = 25 cm, 2β =

$$\beta = \arcsin\left(\frac{HI}{HN}\right)$$

15°4′.

Therefore,
Being much greater than a torsion of $1'$ of the

$$\alpha \approx \frac{2\beta}{10} = 1°30'4'', \;\; 2\alpha \approx 3°0'8''$$

arc, it prevents the formation of closely corresponding retinal points.

Taking into account all of these factors, the geometric locus of the object points that gives rise to corresponding retinal points *is not* the horopteric circle of Vieth-Müller. The horopter passes by the fixation point, but as the eccentricity of the retina increases, the uncertainty regarding its position increases proportionally. As Hermann von Helmholtz (1821–1894) noted in his visual analysis of frontality in three pins arranged on a wooden ruler 50 cm from the eyes, "The most favorable case is always that in which the direction of the line of pins corresponds to the direction of the tangent to the horopteric circle." [24D] The conclusion drawn by the nineteenth-century physiologist was therefore identical to the one arrived at by Ibn al-Haytham based on his experiments.

On these grounds one can understand more clearly why Ibn al-Haytham, who was analyzing this problem from the perspective of an experimentalist rather than a geometer, retained the frontal line TQK as the definition of the geometric locus of object points that are seen as single images by the two eyes. The later construction of the horopteric circle is therefore based on dubious hypotheses that do not stand up on closer examination. Nevertheless, in order to grasp the rationality of Ibn al-Haytham's concept of the frontal horopter, it is necessary to study the history of the works from which the determination of the experimental horopter sprang.

Panum's fusional area (Experiment 5)

Taking as his departure point a critique of the Vieth-Müller circle, in 1858 Peter Ludvig Panum proposed a new conception of retinal correspondence. [50, 12] The Danish physiologist had set out to study the degree of disparity that could be tolerated when two images are fused in binocular vision and he devised the following experiment. Given the two eyes C_L and C_R and the fixation point F, one places two small vertical bars B_1 and B_2 in proximity to F, in such a way that for one of the two eyes (say, the left eye), they are aligned along

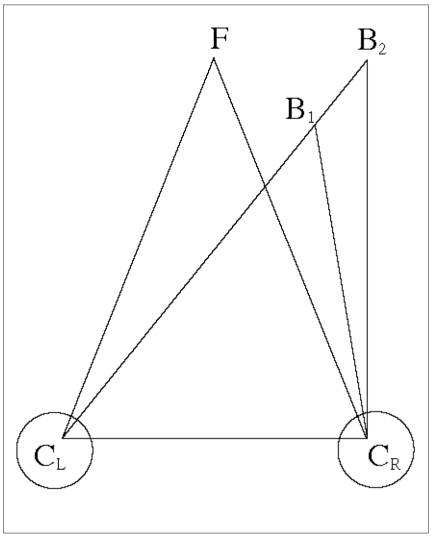

Figure 7
Panum's experiment.

Author's drawing.

the axis $C_L B_1 B_2$. By displacing B_1 along the visual axis $C_L B_2$, one searches for the interval in which the barrettes are seen as single images by the other eye C_R (*Figure* 7).

One then determines the external and internal limits (corresponding to homonymous and heteronymous diplopia, respectively) of the area containing the object points that are seen as simple images in binocular fusion.

This is Panum's fusional area, reflecting the fact that retinal points which do not correspond exactly but are slightly disparate can nevertheless be fused into a single image. [24E]

Kenneth N. Ogle [48] took up where Panum left off, devising an experiment in which the fixation point was situated 40 cm from the eyes. In this way he succeeded in determining the position of Panum's fusional area between the two arcs corresponding to the thresholds of homonymous and heteronymous diplopia (*Figure 8*).

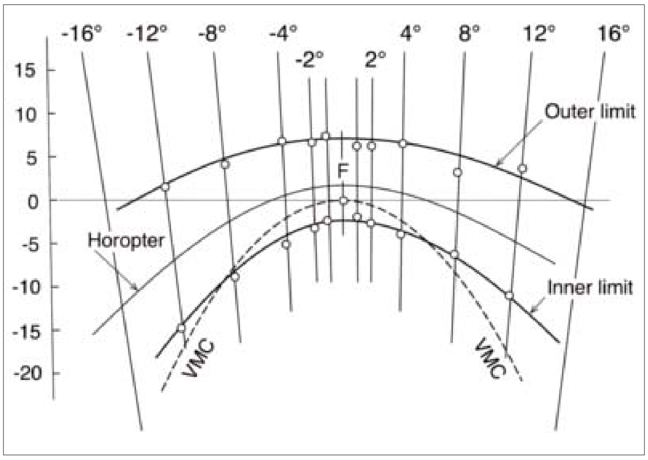

Figure 8
The horopter and Panum's fusional area.

Author's drawing after Ogle, *Researches in Binocular Vision*, 1964.

Ogle's abacus represents the averages of the internal and external limits – which fix the thresholds of homonymous and heteronymous diplopia – together with the Vieth-Müller circle (marked *V-M* in the figure) which diverges from Panum's fusional area with an eccentricity of more than 6°. The horopter, which from now on would be defined as the arc situated midway between the external and internal limits, satisfies the conditions for the conic section:

(given $u_1 = FC_LM$ and $u_2 = FC_RM$, using

$$\text{cotan } u_1 - \text{cotan } u_2 = H$$

the same notation as in *Figure 4*). As Ogle writes, the real horopter is "a curve set halfway between the Vieth-Müller circle and the tangent in *F* to this circle." [79A] In other words, for distances on the same order of magnitude as the length of Ibn al-Haytham's wooden ruler, the Vieth-Müller circle provides an upward biased model of the retinal correspondence, whereas the frontal line provides a downward biased model. The curvature of the experimental horopter changes with the distance from the fixation point. For short distances the horopter is concave. For the so-called "abathic distance," [37C] which varies from one to six meters as a function of the distance between the pupils of the two eyes, the horopter coincides with the frontal line that passes through the fixation point. Beyond the abathic distance, the experimental horopter becomes convex and has only one point of tangency with the Vieth-Müller circle.

These elements allow us to formulate a last question regarding the horopter. Could the frontal model chosen by Ibn al-Haytham simply be the result of the fact that, if one uses a sufficiently narrow ruler for the experiment (TK = 6 or 7 cm), the frontal line and the horopteric circle are virtually merged? On the experimental ruler of Ibn al-Haytham, Q is the point of fixation. The Vieth-Müller circle passes through K' near point K, and through T' near point T. Q' is the point of intersection of the middle line HQ and the front line $T'K'$. QQ', which is the sagitta of the circle, can be calculated: $TT' = KK' = QQ' = R$ $(1 - \cos \alpha)$. Since R = 12.75 cm and $\alpha = 14°$ $30'$, it follows that the maximum distance between the two appears along the edge of the ruler: $TT' = KK'$ = 0.41 cm. Are we therefore in Panum's area, where the images of points K and K' fuse? Ogle's abacus yields, for a fixation point located 40 cm from the eyes and an eccentricity of 8° (angle QHK = 7° 28′), a tolerance of 4.6 mm (*Figure 8*). However, the closer the fixation point, the lower the tolerance. As a consequence, in Ibn al-Haytham's experiment the tolerance should be much less than 4.6 mm and the points K K' should not be seen as fused. Repeating Ogle's experiment using Ibn al-Haytham's binocular tablet shows that a pin K' positioned at 4 mm in front of K is effectively seen as doubled. These experiments were reproduced with my replica of the instrument, on display at the Galleria degli Uffizi (Florence) from October 16, 2001 to April 7, 2002. [61] In Helmholtz's experiment, which consisted of moving three sliding rulers to which pins were affixed, using the middle pin as the fixation point, the author noted that it sufficed to move the lateral pin 'a half-pin's width' (0.25 mm) to see a double image. [24D] Without attaining his degree of precision, we frequently observed a doubling of the image between 1 and 2 mm. One must therefore not accept the argument that the line TQK represents an approximation of the theoretical horopter, because the two would be virtually identical. For the observer, the circular horopter and the frontal horopter are *not* identical.

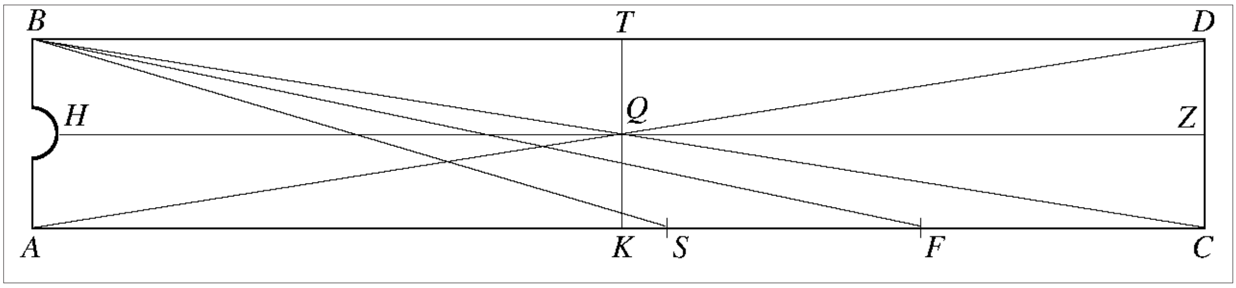

Figure 9
Ibn al-Haytham, Experiment 5.

Author's drawing.

It may be concluded that Ibn al-Haytham adopted the hypothesis of the frontal line for a completely different reason than the one traditionally ascribed to him; from an experimental point of view it was more convincing than the model based on a circle. In fact, the object points of the frontal line *TQK* fall into Panum's fusional area, whereas the object points of the circle *T'QK'* are seen as double images.

Let us return now to Panum's fusional area and the question of the approximate correspondence of images. While the experiments designed by Panum and Ogle were entirely original, the notion of measuring the degree of disparity tolerated in fusion cannot be attributed to them. In fact, one finds a first expression of this in Ibn al-Haytham's *Kitāb al-manāẓir*.

Experiment 5. [31D] The columns of wax being lined up once again in positions *T Q K* and with the eyes fixed on *Q*, take the column in position *K* and move it along the side of the ruler *KC*. Close to *K*, at point *S* the column will still be seen as a single image. Beyond this point, for example at *F*, the column of wax will be seen as double (*Figure 9*).

This observation – which remained qualitative in nature – did not allow Ibn al-Haytham to determine the extent of the space within which the object points were fused. Nonetheless it paved the way for Panum's notion of the fusional area because, by displacing the column of wax along *KC*, he clearly established that fusion also operates for object points that are not *strictly* contained within the horopter. It would have sufficed for Ibn al-Haytham to repeat the experiment, moving the column of wax along *KC*, *TD*, *QZ* and many other parallels to these lines, to arrive at a definition of the tolerances allowed by correspondence.

Ibn al-Haytham's doctrine can be summarized as follows: (1) objects will be seen as single images if they are arranged in corresponding or almost corresponding directions, that is to say, if they lie in the frontal plane that passes through the point of fixation (in modern terms, the horopter); (2) objects will be seen as single images if their position does not deviate excessively from the frontal plane (in modern terms, Panum's fusional area). If these conditions are not met, the objects will be seen as double images. Diplopia was therefore studied as a general physiological process.

Ibn al-Haytham's legacy

The legacy of Ibn al-Haytham's optics in the Latin world will be studied in two parts. The first will focus on the diffusion of his work and on the availability to scholars of manuscript copies and printed editions of his texts. Since the existence of a text did not necessarily mean that it was read or its contents accepted, we will seek in the second part to clarify the conditions under which the binocular theory of vision might have provided an impetus for the work on perspective that took place between the Duecento and the Cinquecento.

As has been shown in the first section of this article, the research on optics that began in the seventeenth century (on topics such as the theory of corresponding points, direct and crossed diplopia, the construction of the horopter, and early notions of the fusional area) cannot be understood without an assessment of the work of Ibn al-Haytham, which laid the foundations for many of the discoveries of modern physiological optics. One can detect here a stratification of scientific concepts comparable to that seen in the studies of refraction undertaken by Ptolemy, Ibn Sahl, Ibn al-Haytham, Harriott, Snell and Descartes. Although the savants of the classical period did manage to shed considerable light on how vision functioned by explaining the role of the crystalline lens and

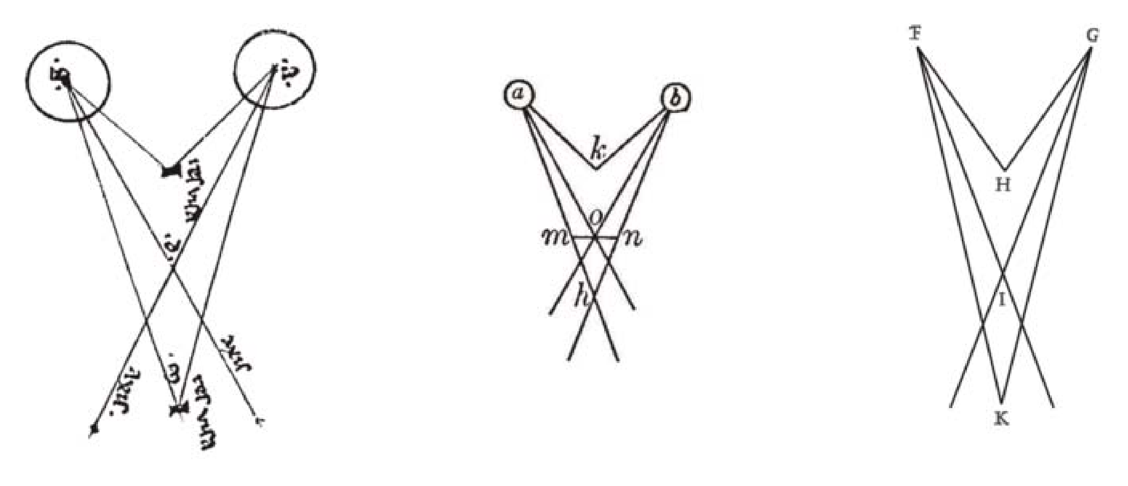

Figure 10
The legacy of De aspectibus, prop. III, 2.9: (A) Alhacen, Paris, BnF lat. 7319, fol. 116v (Permission of the BnF); (B) Bacon, Perspectiva (ed. JH Bridges, The 'Opus majus' of Roger Bacon, II, p. 95); (C) Pecham, *Tractatus de perspectiva*, (ed. DC Lindberg, p. 73).

the retina, it must be recognized that their understanding of the conditions for the fusion of binocular images was still rooted in medieval theories of optics. On this point there was no genuine breakthrough before the discovery of the neuro-physiological foundations of binocular vision.

To what may be ascribed the impression of a marked lack of scientific progress before the seventeenth century? An examination of how the science of optics was received allows us to formulate an explanation. In classical Europe, where translations and commentaries on most of Ibn al-Haytham's work on optics were in circulation and discussed, [40, 77B] savants might have believed the study of binocular vision to be so *saturated* with commentary that it was enough to cite the accepted authorities with no need for further analysis. Herein no doubt lies one of the main reasons for the mistaken perception that the problems of physiological optics were discovered *ex novo* in the seventeenth century.

Let us now examine the revival of Ibn al-Haytham's ideas on optics in more detail. His work was known through a translation from the Arab to Latin that provided the basis for the edition published by Risner in 1572. It also spread through the many commentaries written by medieval and Renaissance scholars. In particular, it is known that Ibn al-Haytham's experiments on binocular vision were repeated and abundantly discussed in the Latin world during the entire course of the Middle Ages.

If Risner's edition did not always provide a literal translation of the original text, the essential elements of the theory laid out in *Kitāb al-manāẓir* could be found in *De aspectibus*, a twelfth-century Latin translation of Alhacen's work, whose author is unknown but who probably belonged to the circle of Gerard of Cremona. The Latin text can be attributed to two successive hands. The translation is quite literal up to Book III, Chapter 3, before becoming looser and sometimes resembling a paraphrase more than a translation of the original Arabic. The passages on binocular vision appear in III, 2 and therefore were the work of the first translator. Even if the presumption is that the Latin version came from Spain because of the voiceless 'c' in Alhacen appearing in the earliest manuscripts [41, 69D] its attribution to Gerard of Cremona remains questionable [77D] This text served as the source for a translation of Alhacen's work into the vernacular, *De li aspecti*, which was in all likelihood compiled in northern Italy in the middle of the fourteenth century. Philological analysis has shown that MS. London, British Library, Royal 12.G.VII served as the Latin matrix for the Italian version. [75] This text, discovered by Enrico Narducci [45] was investigated by Graziella Federici Vescovini. [19, 20] This Italian edition constituted a genuine milestone that is crucial to our understanding of the spread of Ibn al-Haytham's ideas in medieval and Renaissance Italy. *De li aspecti* transmits *in extenso* the chapters dedicated to diplopia and the fusion of quasi-images (Vatican, Vat. lat. 4595, fols. 57r–66r).

Not only could Alhacen's text be read in Latin or Italian; it also formed the subject of

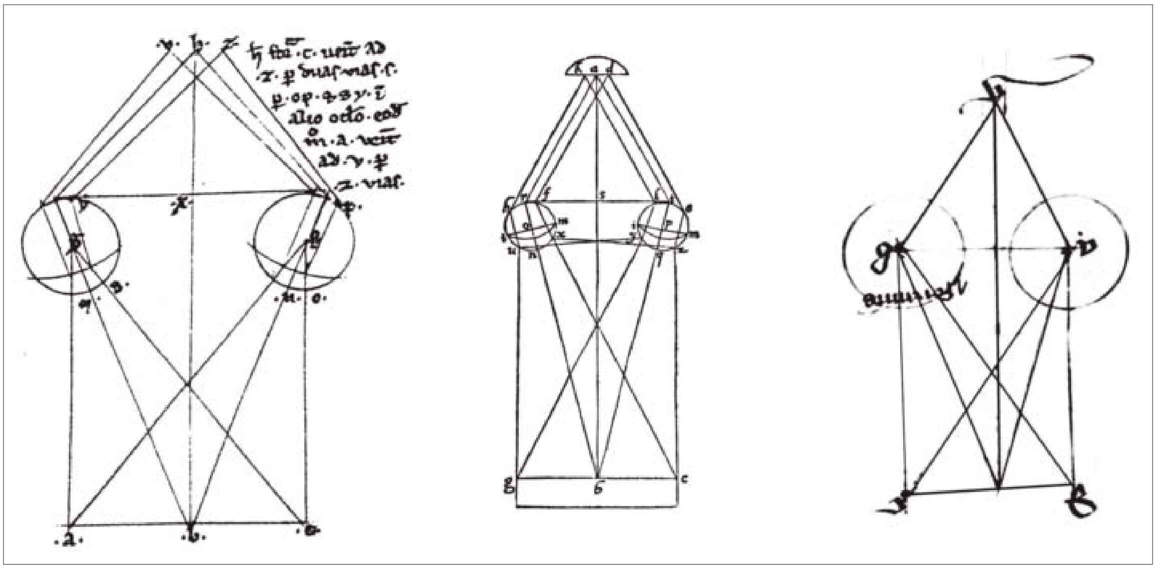

Figure 11
The legacy of De aspectibus, prop. III, 2.15: (A) Alhacen, Paris, BnF lat. 7319, fol. 117r (Permission of the BnF); (B) Witelo, Perspectiva (ed. F Risner, Opticae Thesaurus, p. 103); (C) Pecham, Perspectiva communis (Paris, BnF lat. 7368, fol. 15r).

many scholarly commentaries, [11, 43] and was mainly echoed in the *Perspectiva* of Roger Bacon; [6] Witelo's work of the same title; [88] *Tractatus de perspectiva* [53] and *Perspectiva communis* by John Pecham; [52] and *Questiones super perspectiva communi* by Biagio Pelacani da Parma. [51] It may be noted that the aims of each of these authors differed; Bacon elaborated a scholastic interpretation of Alhacen's theories, whereas Witelo presented a faithful paraphrasing, and Pecham condensed his ideas into a succinct aide-mémoire. Nevertheless, they all copied Alhacen's diagrams and arrived at the same conclusions, albeit sometimes in more simplified form. These similarities become apparent when one compares the texts and figures on some of the more important propositions regarding binocular vision.

Prop. III, 2.9. Here Alhacen distinguishes between the two cases of diplopia:

> Heteronymous (crossed) diplopia: "When that other visible object lies nearer both eyes than the visible object on which the two [visual] axes intersect / *quando illud aliud visum fuerit propinquius ambobus visibus viso in quo coniunguntur duo axes;*
> Homonymous (direct) diplopia: "When that other visible object… lies farther from both eyes than the visible object on which the two [visual] axes intersect / *quando illud aliud visum… fuerit remotius ab ambobus visibus viso in quo coniunguntur duo axes.*

Bacon and Pecham both copied Alhacen's diagram (*Figure 10, p.139*) and came to the same conclusion: the fixation of a central point induces a doubling of the points in front of or behind it. Even so, they differed in their understanding of the problem. Roger Bacon was pursuing the same objective as Alhacen – "In which it is shown… how a single thing appears double / *In quo ostenditur… quomodo unum videatur duo*" – but he cut short the discussion of the figure in order to present the results of his own experiments using a binocular ruler. For his part, John Pecham interpreted binocular vision as a form of optical illusion (*errore, deceptio*) attributable to a defect in the vision or to limitations in the visual conditions, [53] which actually constitutes a misinterpretation of Alhacen's text.

Prop. III, 2.15. In this proposition Alhacen explains the fusion of two quasi-images in terms of the existence of corresponding points: "So the two forms impressed on the two points that are correspondingly situated with respect to the surfaces of the two eyes reach to that same point in the hollow of the common nerve, and they will be superimposed at that point so as to produce a single form / *Et due forme qui infiguntur in duobus punctis que sunt consimilis positionis apud superficies duorum visuum perveniunt ad illum eundum punctum concavitatis communis ipsius nervi, et superponentur sibi apud illum punctum, et efficientur una forma.*" [77D]

Latin authors such as Witelo and Pecham copied this figure (*Figure 11*). In his text Witelo

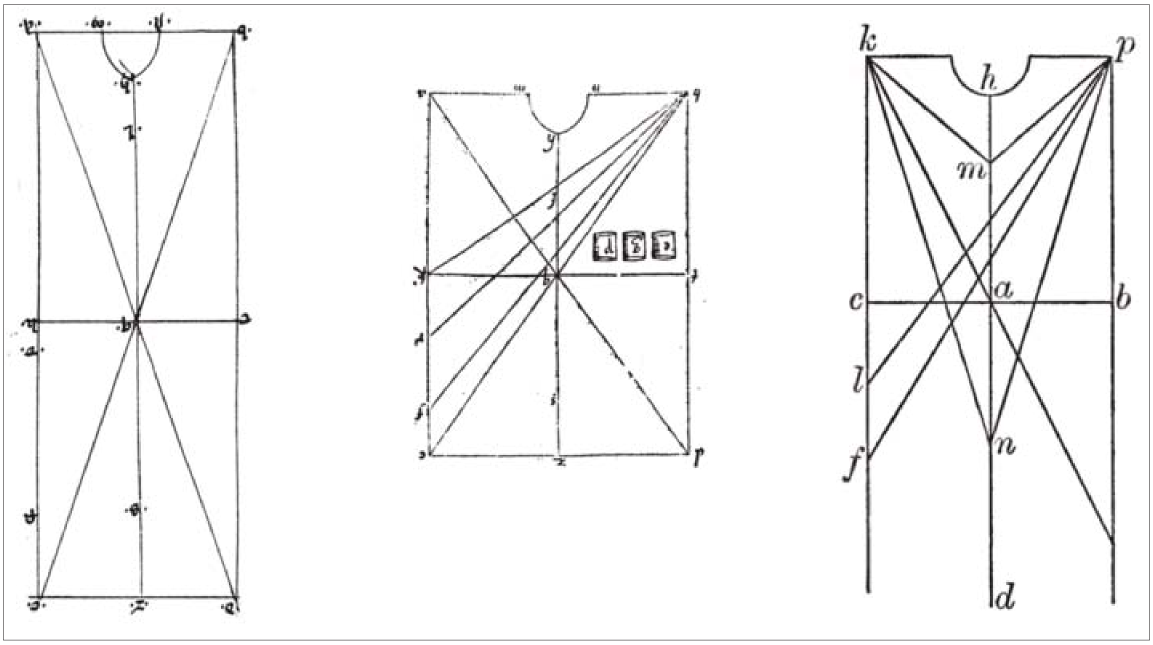

Figure 12
The legacy of De aspectibus, prop. III, 2.27: (A) Alhacen, Paris, BnF lat. 7319, fol. 121v (Permission of the BnF); (B) Witelo, Perspectiva (ed. F Risner, Opticae Thesaurus, p. 165); (C) Bacon, Perspectiva (ed. JH Bridges, The 'Opus majus' of Roger Bacon, II, p. 96).

repeats Alhacen's argumentation, paraphrasing essential passages such as his notion of corresponding points (in two points correspondingly situated / *in duobus punctis consimilis positionis*). John Pecham simplifies the Arab scholar's text, arguing that vision is actually quite straightforward because, barring anomalies and accidents, the images in binocular vision are always fused at the level of the optic nerve: "The duality of the eyes must be reduced to the unity / *Oculorum dualitatem necesse est reduci ad unitatem*." [52B] He schematicizes the argument by discussing the propositions on binocular vision separately (*Perspectiva communis*, I, 32 et I, 80). He does not in fact make a distinction between the two types of physiological diplopia nor does he explain the disparity in the images stemming from binocular vision and their non-corresponding positions in the two eyes until proposition I, 80 (An object appears double when it has a sensibly different position relative to the two axes / *Ex variatio sensibiliter situ uisibilis respectu duorum axium ipsum duo apparere*). Recognizable here nevertheless – apart from a few differences in terminology – is the same explanation that was provided by Alhacen, according to which quasi-images will fuse if they reach corresponding points in the two eyes (impressed on the two points that are correspondingly situated / *infinguntur in duobus punctis que sunt consimilis positionis*), an argument that would be used later by many authors, including Huygens and Müller.

Prop. III, 2.27. In this proposition Alhacen describes the wooden tablet that served for his experiments on binocular vision. Witelo and Bacon copied his figure, each adding the lines that illustrated the results of his experiments, whereas the drawing in *De aspectibus* simply showed the ruler with its pins (*Figure 12*).

Witelo used the same tablet (a board of one cubit… and four digits / *tabula… unius cubiti… quattuor digitorum…*) and presented the same conclusions, slightly condensed, as those laid out by Alhacen (then the experimenter examines…, the experimenter fixes the visual axes… if the experimenter directs his visual axes… / *deinde experimentator inspiciat…, experimentator figat axes uisuales…, quod si experimentator dirixerit axes uisuales…*). Roger Bacon only retained the results of Alhacen's principal experiments (nos. 1, 3 and 5), which he presented in a series of succinct propositions. He added that one could arrive at the same conclusions without resorting to the experimental ruler (Even without such a board, an experimenter can test many things relevant to these matters / *Et experimentator potest sine tabula experiri multa in hac parte*).

While it is clear that the authors of the Latin Middle Ages broadly adopted the teachings of Alhacen (with some modifications and distortions), it may be noted that the influence of *De aspectibus* continued throughout the Renaissance.

It appears that *De aspectibus* achieved such undisputed status that it could not be ignored by any scholar embarking on the study of binocular vision, and many authors continued to copy or paraphrase the same passages. One finds lengthy extracts in the *Commentario terzo* of Lorenzo Ghiberti, [22, 7] who reproduces Alhacen's chapter on binocular vision almost in its entirety (i.e. props. 2.1–2.13, 2.19–2.47, 2.51–2.65 an 2. 71–2.86, following Smith's numbering) and in many of the notes of Leonardo da Vinci. [83] During the Cinquecento binocular vision was discussed by Girolamo Cardano in *Problematum medicorum* [9] and by Vignola in *Due Regole della prospettiva pratica*. An edition of the latter with commentary was later published by Egnatio Danti. [13] On 23 December 1591 Christoph Grienbergen delivered a lecture at the prestigious Collegio Romano [23] in which he broached the question of binocular vision beginning with a presentation of the theories of Alhacen and Witelo. Alhacen was also cited by the eminent scholar Giambattista della Porta in 1593. [14A]

This collection of texts – from translations of *Kitāb al-manāẓir* into Latin and thence into Italian, to Latin commentaries and supercommentaries – provides a picture of the impact of the theories of Ibn al-Haytham on the conceptions of binocular vision in the Latin world. With regard to the accessibility of his texts, it can be taken for granted that the perspectivists had the opportunity to borrow elements from the *Optics* of Ibn al-Haytham in either the Latin or Italian versions. But did they do so? Before addressing this question, let us examine the compatibility of his theory with the problem of representations in perspective.

Obstacles to the binocular theory of vision

When the theory of binocular vision was applied to perspective it met with objections that we will now pass in review.

In the literature one finds various misinterpretations of the functioning of binocular vision, which tend to hierarchize fusion and diplopia, considering simple vision to be the normal mode of seeing and diplopia a marginal condition that might occasionally develop. Since fusion provided a more intuitively convincing explanation of perspective than diplopia, the postulate of monocular vision appeared to be justified. Closer analysis shows that this hierarchy, overtly affirmed in eighteenth-century treatises on optics, [57] is not consonant with the theories of Ibn al-Haytham for it rests on anachronistic notions and simplifications that must be understood if we are to reconstruct the context in which Ibn al-Haytham and his Latin successors conducted their research.

As it is now known that Ibn al-Haytham's theory of binocular vision circulated widely in two versions – Latin and Italian – we will henceforth cite passages from both as we review the development of optics in the Latin West.

Physiological and pathological diplopia

Since Book III of Ibn al-Haytham's *Optics* was devoted primarily to the study of optical illusions (*deceptiones visus*), one might be drawn into believing that the theory of binocular vision could be reduced to a theory of pathological diplopia. But Ibn al-Haytham's treatment of this theory is limited to chapter 2 of Book III, which forms an introduction to the study of optical illusions presented in chapters 3 to 7. The summary to Book III stipulates this clearly:

> The second [concerns certain] things that need to be set forth for the analysis of visual illuions/ *Secundum de eis que debent proponi sermoni in deceptionibus visus* / *El sicondo de quelle cose che se debano propore sicondo tuto in le deceptione del uiso*. [77E, Vat. lat. 4595, 57rb]

The closing paragraph of the chapter (prop. 2.86) is no less explicit:

> And now that these points have been explained, it is time to begin the discussion of visual illusions and to describe their causes and their kinds / *His autem declaratis, incipiendum est de sermone de deceptione visus et declarare causes et species earum* / *Dechiarato questo e da guadare in lo sermone dela deceptione del uiso e dechiarare de cagione e le spetie de esse*. [77F, Vat. lat.4595, 66ra]

As a consequence, it was actually within a general framework that Ibn al-Haytham undertook his study of the conditions for the fusion of quasi-images. This point has not always been clearly grasped by historians of science, who have tended to present the conclusions drawn by Ptolemy and Alhacen from their binocular experiments as follows:

> In strabismus, most often one of the eyes plays a dominant role that for all practical purposes overrides the contribution of the other; however, the paralysis of an oculomotor muscle, or simply pressure on one of the eye globes, allows the formation of two independent retinal images, thus causing double vision or diplopia. [73]

This reading does not distinguish between physiological and pathological diplopia, and treats all cases of double vision as abnormal. In contrast, the examples of diplopia discussed in chapter III, 2 represent illustrations of normal physiological diplopia and have no particular relationship to strabismus, pathological diplopia, or the phenomenon of optical illusions. Further confirmation is provided by three passages in *De aspectibus* which interpret mistaken perceptions of size as a consequence of monocular vision; see prop. 2.51, 2.52 and 2.53.

Diplopia is not an unusual phenomenon
The fact that chapter III, 2 opens with a discussion that takes simple vision as the standard case allows one to deduce that diplopia occupied a secondary position in Ibn al-Haytham's theory of optics. For example, his first proposition poses the question: "Since it is so, we must determine how a single, distinct object is seen simultaneously by the two eyes as one image, *most of the time and in most cases*, and how it happens that a unique object can be identically situated in relation to the two eyes, *most of the time and in most cases.*" [69E] The Latin and Italian versions of his text read:

> Prop. 2.1: So we need to explain how a single visible object is perceived as single by both eyes [*most of the time*] and in many [different] situations, as well as how the situation of a single visible object will generally be equivalent with respect to both eyes under various conditions / Unde oportet nos declarare quomodo unum visum comprehenditur a duobus visibus unum *in maiori parte temporis* et in pluribus positionibus, et quomodo positio unius visi ab ambobus oculis *in maiore parte temporis* et in pluribus erit consimilis / Doue fia di bisogno noi dechiarare como uno uiso zioe como una cosa ueduta si comprendre da dui uisi como el uno *in magiore parte di tempi* e in piu dispositione e chomo la positio de uno seno (?) da amedui lochij *in magiore parte del tempo* e in piu sera consimille. [77G, Vat. lat. 4595, 57vb]

All the same, the phenomenon of diplopia is not rare. Ibn al-Haytham acknowledges this, for example, in prop. 2.9, which introduces the two cases of homonymous and crossed diplopia, and by prop. 2.44 and prop. 2.45, which describe how the lines on his ruler are perceived by the two eyes:

> Prop. 2.9: Furthermore, the axes of both eyes *often* intersect on some visible object while the two eyes perceive another visible object that is not correspondingly situated with respect to the eyes in terms of direction… / Et *multotiens coniunguntur* duo axes amborum visuum in aliquo viso, et cum hoc duo visus comprehendent aliam rei visam cuius positio in respectu duorum visuum erit diversa in parte… / E *molte uolte se congiungeno* le axe de tuti dui li uisi in alchuna cosa uisa e cun queste dui uisi comprendeno comprenderano laltra cosa uisa dela quale la positione in rispecto de amedui i uisi sera diuersa in la parte. [77H, Vat. lat 4595, 59ra]
>
> Prop. 2.44: Yet this line, and everything that lies on it, except for the peg that is placed in the center, *invariably* appears double if the two [visual] axes intersect at the peg placed in the center / Et ista linea et omnia posita super ipsam preter individuum positum in medio *semper videntur duo* cum duo axes concurrerint in individuo posito in medio / E questa linea e tute quelle cose che sono posite sopra quella ultra lindividuo posito in lo megio *sempre pareno due* quando due asse serano concurse in lindividuo posito in lo megio. [77I, Vat. lat 4595, 62va]
>
> Prop. 2.45: On this basis it has been therefore been shown that a visible object that lies on different sides of the two [visual] axes *always* appears double / Declaratum est igitur ex hac dispositione quod visum cuius positio in respectu duorum axium est diversa in parte *semper videtur duo*… / Dechiarato e adunche da quella dispositione che quello che si uede quando la positione de esso in rispecto de due asse e diuersa in parte *sempre pareno due*. [77J, Vat. lat 4595, 62va]

The adverbs "often" (*multotiens*) (2.9) and "always" (*semper*) (2.44, 2.45) preclude limiting diplopia to exceptional cases of abnormal vision. Therefore, the meaning of passages in which Alhacen describes cases of simple vision as "frequent" must not be misinterpreted. Objects are seen as single images if they lie in corresponding or almost corresponding directions. In all other cases they will be seen as double images.

Images are not blurred in diplopia
Another interpretation that permits the hierarchization of simple and double vision, with simple vision being considered as normal and double vision as the exception, is based on the argument that diplopic images would be out of focus. In fact, all retinal images tend to be blurred around the edges because the conditions of stigmatism are no longer met and the cones, which are more sensitive to light, are less numerous toward the periphery. This argument cannot be found in *Kitāb al-manāẓir* because the anatomical knowledge on which it is based was discovered much later. Here, for example, is what the Latin and Italian versions of Alhacen's text have to say about those cases in which quasi-images are disparate:

Prop. 2.19: Nevertheless, its form will be indefinite rather than definite / *Sed tamen forma eius non erit verificata sed dubitabilis / Ma niente meno la forma de esso non sera certificata seno dubitabile.* [77K, Vat. lat 4595, 60va]

Prop. 2.20: So the form of its extremities will be indefinite rather than definite / *Quapropter forma extremorum erit dubitabilis, non certificata / Per la quale cosa la forma degli estremi essi dubitabili o uoi sera dubitabile, non certificata.* [77L, Vat. lat 4595, 62vb]

Prop. 2.21: And so the form of such visible objects will be indefinite under all circumstances… / *Et sic forma huiusmodi visibilium erit dubitabilis in omnibus positionibus… / Cosi la forma de questi uisibili sera dubitabile in tute le dispositione.* [77M, Vat. lat 4595, 61ra]

Although the fusion of corresponding points leads to "definite" vision in the sense that it is certified by the organ of sight (*certificata, verificata*), diplopia does not produce a blurred image. The entire argument here rests on the word *dubitabilis* – "dubious, what may be doubted" – which brings one back to the duality of the quasi-images produced by the two eyes. However imperfectly superimposed, these quasi-images do not in the end merge in a synthesis that "glues together" their salient traits. It is precisely because they are clearly distinct that the eye hovers in doubt as to what it is seeing; that is, it does not know which of the two images it should choose.

Different theories regarding the unification of binocular images

The theory of the *suppression* of one of the two images in binocular vision and the critique of this theory, which will be examined further below, can only be understood within the framework of the notions that were conceived in order to account for the unification of the visual sensations received by the two eyes. [84, 86] These theories can be divided into three groups: the theory of permanent fusion, the theory of conditional fusion, and the theory of the absence of fusion.

1) Permanent fusion

According to the earliest of all the theories, that of permanent fusion, the images received by the two eyes are always associated and joined together. This was proposed most notably by Pseudo-Aristotle (*Problemata* XXXI, 4, 7), who retained that the functioning of the two eyes could not be disassociated, either in terms of motor function or of perception. Is it not, as he asked, "[…] because the eyes, although two in number, depend on one and the same principle?" [4] This theory would be adopted by John Pecham who, considerably simplifying the text of Alhacen on this point, wrote: "*The duality of the eyes must be reduced to unity / Oculorum dualitatem necesse est reduci ad unitatem.*" [52B]

The considerations of Galen (*De usu partium* X, 14) and Pseudo-Aristotle (*Problemata* XXXI) survive in a question in *Problematum medicorum* by Girolamo Cardano. The precise date of this text is not known because it was never published during the author's lifetime; [47] all that can be affirmed is that the work was written before the death of Cardano in 1576. Question 3 reads:

> Why does the myopic see better with both eyes than with either one of the two eyes, while those who have good vision see as much with a single eye?—the proof is that they close one eye when they want to aim [their vision] in a straight line. But rather than aiming, would it not be better just to see? Is it that, on closer inspection, the visual species of a single eye is more concentrated in the cone? I also state that the myopic only sees a portion [of the visual field] with each eye, the other part is not seen, and so the two [eyes] are needed to see everything. The remaining [visual power] helps the operation of the other eye. Suppose that *K* sees *ABCD* straight ahead and *BDEF* on the other side; that *L* sees *BDEF* straight ahead and *ABCD* on the other side. Nevertheless *K* and *L* will perfectly see all parts, and all the better the whole that *K* [will see] his half *ABCD*, and *L* his own *BDEF*. But for perfect vision, the [visual] power of the closed eye combines with that of the other [eye], and as a result, he sees clearly; and rather better when he looks with one eye rather than two [eyes]. Why, when the organ is injured and corrupted, the strong (for example, that having strong fire from a little water) is easily strengthened? Is it because he

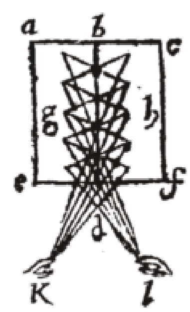

Figure 13
The theory of binocular vision according to Cardano, Opera Omnia, tomus secundus, p. 640.

> gathers his power, or is this not always true? Where the light is lacking, as in old men, light added to the other eye allows them to see better, while those who are filled with [light] are prevented from doing so. This is why the same [old men], because they see better at a distance, as far as they are brought by nature, see quite well with both eyes. [9] (*Figure 13, p.144*)

This text is surprisingly anachronistic. Cardano revives the theory of extramission, which was abandoned by most medieval scholars after their exposure to the work of Alhacen. It also illustrates the difficulty that Cardano had in choosing between the conflicting theses of Pseudo-Aristotle; visual fixation is better using one eye, but visual perception is clearer using two eyes (*Problemata* XXXI, 4 and 10). Cardano attempts to reconcile the two notions by proposing a less than satisfactory compromise – allowing for fusion in the case of myopathy and suppression in the case of normal vision.

The theory of permanent fusion would then be taken up by Egnatio Danti who, citing Risner's edition of the works of Alhacen and Witelo, [13B] kept nothing of the theory of correspondence that formed the core of the analysis of binocular vision. The doctrine of permanent fusion would reappear in Descartes' *Dioptrique*. [15A] Published in Leiden in 1637, *Dioptrique* ignores the teachings of Aguilonius and Scheiner which were published in 1613 and 1619. According to Descartes, the unification of visual sensations is controlled by a small gland which he denominated the *glandula pinealis* or *conarium*. In reality this gland is responsible for the synthesis of melatonin, but Descartes believed it to be "the seat of common sense." In the sixth Discourse he reiterates his thesis by drawing the famous parallel:

> And as the blind man does not judge a body to be double although he touches it with his two hands, so too, when both our eyes are disposed in the manner required to direct our attention to one and the same place, they need only make us see a single object there, even though a picture of it is formed in each of our eyes / *Et comme cet aveugle ne juge point qu'un corps soit double, encore qu'il le touche de ses deux mains, ainsi, lorsque nos yeux sont tous deux disposés en la façon qui est requise pour porter notre attention vers un même lieu, ils ne nous y doivent faire voir qu'un seul objet, nonobstant qu'il s'en forme en chacun d'eux une peinture*. [15B]

2) Suppression

Permanent fusion represents a single theory of vision, whereas the thesis of non-fusion allows for two distinct types of vision in which a monocular image may be generated by one eye or by the two eyes alternately. The sources of the theory of suppression are quite ancient; they can be traced back to Galen (*De usu partium* X, 14), who espoused the theory of extramission and believed that the pneuma descended from the encephalon and was shared by the two eyes in binocular vision, whereas it was transmitted integrally to one eye in monocular vision. From this Galen drew the conclusion that monocular vision was more acute than binocular vision. [21B] Some of his successors took the argument to its extreme and claimed that, the two eyes being intact, the processing of images was conducted monocularly.

Case 1. The image is furnished by one of the two eyes. The best known version of this thesis is probably the one proposed by the Jesuit scholar Giovanni Battista della Porta of Naples, who wrote in *De Refractione*:

> Nature has bestowed on us two eyes, one on the right, the other on the left, so that when we are going to see something on the right, we see with the right eye, and on the left with the left eye… Therefore we always see with a single eye… Hence the two eyes are not able to see the same thing at the same time / *Oculos binos natura largita est nobis a dextris unum, a sinistris alterum, ut si a dextris aliquid visuri sumus, dextro utamur, at si a sinistris sinistro, unde semper uno oculo videmus... Unde non simul videre possunt rem eandem*. [14B]

In 1679 Sébastien Le Clerc, a French engraver who had taken up the study of geometry, wrote a short treatise in support of the thesis of suppression. [35] Mistaken ideas are often slow to be abandoned; Le Clerc drew his principal argument not from the works of Della Porta, but from the lines of investigation pursued by Ibn al-Haytham and others. He based his thesis of suppression on the conundrum posed by the disparity between images, finding himself unable to explain how they were unified, even though Aguilonius had already presented a convincing geometric analysis of the problem in 1613.

In the eighteenth century Porterfield [57B] would formulate, on anatomical grounds, a critique of the ancient idea that the unification of visual sensations takes place at the level of the chiasma. The role assigned by Descartes to the pineal gland (which the optic nerves do not actually reach) is ascribed by Porterfield to this crossing point between the two optic nerves. Being aware of the latest discoveries regarding the horopter, Porterfield acknowledges that when images are disparate they are seen as double. Even so, he sought to minimize the consequences of this by hypothesizing that the soul possesses a 'faculty of learning' that allows it to reunify double images. [57C] The following year Du Tour [17] conducted experiments on binocular vision and established that if two images are disparate they are not fused, but rather are seen sequentially. This paved the way for the modern theory of retinal rivalry, according to which two images are perceived alternately if there is a large disparity between them in terms of form or color.

Case 2. The image is furnished by only one eye, generally the dominant one. This thesis was defended by Francis Bacon in a short treatise published in 1627. [5, 85] One encounters it again in the work of Giovanni Alfonso Borelli who, based on his personal observations, concludes that the left eye is always dominant. [8] These notions would be criticized in *La Vision parfaite* by Chérubin d'Orleans, who challenges the thesis of suppression – both alternate (case 1) and constant (case 2) [16] – and then in the *Physical Essay on the Senses* by Le Cat, who observes that vision can vary from individual to individual, with either one eye or the other being dominant, and sometimes with no dominance at all (case 2). [34]

In French it became the custom to refer to these theories by the term "suppression" in cases where inhibition involves only a part of the image seen in one eye, and the term "neutralization" when this inhibition involves the entire image seen in one eye, whereas the English refer to *suppression theory*, which assumes the partial or total inhibition of the summing of binocular images. [79B]

3) Conditional fusion

In comparison to the preceding theses of unconditional fusion and suppression, the theories in the third group appear quite sophisticated. Here again one notes the presence of two types of phenomena: fusion takes place if the direction of the two eyes corresponds exactly or if they correspond approximately.

Case 1. On the condition that the direction of the two eyes corresponds exactly. It is in this group that one finds once again the classic theories of Ptolemy and Ibn al-Haytham and of their Latin successors. Geometry remaining the preferred paradigm in ancient and medieval optics, the notion of correspondence was always interpreted in the strict sense; that is, the images must appear in similar positions: *fī nuqṭatayni mutashābihatay al-waḍ', in duobus punctis… consimilis positionis, in two points correspondingly situated*. Ibn al-Haytham may have ventured slightly beyond this limit by studying the admissible tolerances in the fusion of images, but his observations were not, as we have already said, sufficient to challenge the empirical definition of the fusional area. Therefore, while medieval Latin authors (such as Bacon, Witelo and Pecham) may have been familiar with these problems, they made no real contribution to the study of optics, which had to await the intuitive discovery of the circular form of the horopter by Aguilonius and its systematic definition by Vieth and Müller.

Case 2. On the condition that the direction of the two eyes corresponds, but only approximately. The development of experimental physiological optics led to the adjustment of the theory of the horopteric circle without calling into question the validity of the concept of "correspondence." The first steps were taken by Charles Wheatstone in 1838; [87] with the aid of a stereoscope he showed experimentally that the unification of visual sensations took place as long as the direction of the two eyes diverged slightly, and that this slight discrepancy contributed to the perception of relief or stereopsis (*Figure 14*).

Wheatstone did not specify the fusion intervals and it was not until the work of Panum [50] that these conditions were defined. This would result in the abandonment of the thesis of the Vieth-Müller circle and its replacement by Panum's fusional area.

With regard to this detailed classification of past optical theories, which we have simplified in order to be able to regroup them, modern physiological optics (most of whose theories fall into the third group) distinguishes between three fundamental situations relative to the integration of visual sensations:

1) an exact correspondence between the image points on the two retinas (the theoretical horopter) results in the simple *fusion* of quasi-images;

2) a slight disparity (of less than 1 degree) in the image points on the two retinas (Panum's area) triggers *stereopsis*, i.e., the fusion of the two images with the perception of relief;

3) a marked disparity triggers *physiological diplopia*, a complex phenomenology that includes homotopic diplopia (the superpositioning of two distinct and disparate images), retinal rivalry (disparate images are seen alternately at a rhythm of 2 to 3 cycles per second), suppression (the partial effacement of one quasi-image in favor of the other), and neutralization (the complete suppression of one of the two images). [70C]

Neutralization is not constant

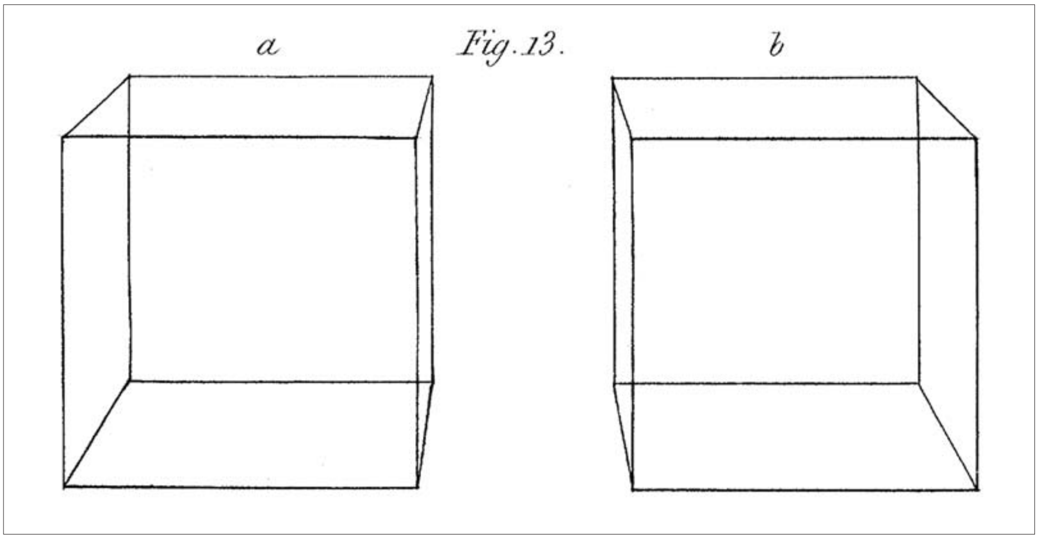

Figure 14
Binocular disparity according to Wheatstone, "Contributions to the physiology of vision," p. 372, Plate XI, Fig. 15.

If we bring together all the historical texts dealing with binocular vision and compare what they have to say on the unification of visual sensations, we find that: 1) the cases of normal and pathological diplopia were regularly confused; 2) the predominant explanation regarding how quasi-images become unified was the thesis of neutralization. Such was the opinion of Albert Lejeune and Gérard Simon, who wrote on the binocular experiments of Ptolemy:

> We always focus on the object to which our attention is given and we unconsciously neutralize everything that could damage the clear perception of what we wish to see. It is true that the observation of double images is very difficult and is only achieved at the cost of much practice. [38B]
> Such redoublings, which should greatly interfere with peripheral vision, generally escape our notice; we often perceive only one of two unfused images, the other is neutralized. [73]

It suffices to consider their observations together with those of specialists in physiological optics to grasp that their works must be read with prudence. Yves Le Grand, who examined the hypothesis of the permanent neutralization of one of the two retinal images, wrote:

> One may have the neutralization of one of the retinal images, which would prevent one from seeing double. This was the opinion of Porterfield (1759), who considered the perception of double images as abnormal or at the very least artificial… Yet, with a little practice, one can perceive very well these double images, as we will show shortly, and the hypothesis of constant neutralization is therefore inadmissible. [37D]

This thesis, which was probably drawn from past sources – including not only Porterfield, but also Della Porta, Gassendi, Tacquet and Le Clerc – does not suffice to explain the functioning of binocular vision for two principal reasons.

Primo, the thesis of constant neutralization would negate the very real benefits associated with binocular vision. Specialists have listed at least ten positive effects, which can be classified in two groups based on the contributions of binocular vision to vision in general and to the perception of three-dimensional space in particular. [79C]

Among its generic properties, it is known that binocular vision: (i) lowers the threshold for the detection of light; (ii) shortens the reaction time to visual stimuli; (iii) increases visual acuity; (iv) increases the threshold of sensitivity to contrast; and (v) increases the sensitivity to spatial contrasts.

Among the properties directly involved in the perception of space, it is known that binocular vision: (vi) facilitates the perception of relief by detecting the curvature of objects in the frontal plane; (vii) allows for the localization of an object in space by concentrating attention on the fixation point; (viii) is responsible for the spatial arrangement of objects through depth perception (Euclid was aware that two eyes could perceive a larger portion of a sphere than one eye alone, *Optica*, prop. 28–30) [33B, 80] (ix) allows one to infer the distance of an object from the fixation point based on the degree of binocular convergence; and (x) generates a mental image beginning with the quasi-images produced by each eye, the most complex phenomenon discussed above. And yet by making the assumption that we see through just one eye at a time, the thesis of constant neutralization denies the existence of all of these properties and in particular, by annulling binocular parallax, suppresses any possibility of the perception of relief and depth.

Secundo, if one accepts the notion that the phenomena of neutralization, suppression and retinal rivalry can be merged, "neutralization" only manifests itself in well-defined situations, as has been illustrated by experiments with black discs or vertical and horizontal grids. One may then question the validity of an extrapolation of the thesis of neutralization beyond these cases to the reduction of double images to single ones.

What was Ibn al-Haytham's position on this point? According to him, simple images are seen when they are situated exactly or approximately in the same frontal plane (the horopter), which corresponds to the case of the fusion of quasi-images. Otherwise what one has is physiological diplopia, that is to say, stereopsis or true diplopia. In the case of fusion, the form of the object is certified (*certificata*); otherwise, the two images are superimposed (*due forme erunt se penetrantes*) and the form of the object is open to doubt (*forma eius non erit certificata sed dubitabilis*). In the follow-up to this analysis, Ibn al-Haytham is led to specify the conditions for the perception of these double images. According to him, we do not notice the doubling of an image if the object is of a single color or texture. Here is the relevant text in three languages:

> Prop. 2.19: Rather, the form of every point that lies far from the point of intersection will be impressed on two points of the two eyes that correspond… And if the visible object is of one color, then the effect of doubling will hardly be noticed because of the correspondence in color and the sameness of the form. If, however, what is seen is multicolored, or if there is some design, or depiction, or [if there are] subtle features in it, then the effect of doubling will be noticeable… / *Sed forma cuiuslibet puncti remoti a puncto concursus figetur in duobus punctis amborum visuum… Et si visum fuerit unius coloris, tunc illud fere nichil operabitur in ipsum propter consimilitudinem coloris et ydemptitatem forme. Si autem visum habuerit diversos colores, aut fuerit in eo lineatio, aut pictura, aut subtiles intentiones, tunc illud operatur in ipsum… / Ma la forma de zi-ascheduno punto rimoto dal punto del concorso se fichara in dui punti de amedui i visi… E sel viso fosse de uno colore alora quello apena operarebe in esso e quasi nulla operarebe per la similtudine <dela forma e dela identi> del colore e dela identita dela forma. Ma se el viso havesse havuto diversi colori o pictura o intentione subtile alora questo opera in esso.* [77N, Vat. lat. 4595 60vb]

The doubling of images is perceptible then, if at least one of the above conditions is met: i.e., the object must be multicolored, or present a complex outline or texture. In all other cases the doubling will pass unnoticed. Ibn al-Haytham's solution offers the advantage of not relying on the theory of constant neutralization and therefore constitutes an important milestone in the development of modern optical theory.

Conclusion

A perusal of the literature on optics shows that the theory of Ibn al-Haytham formed the subject of learned commentary from the Middle Ages to the classical period. It can be concluded therefore that his work was generally accessible to the scientific community in Latin Europe. Physiological diplopia was not regarded at the time as an abnormal phenomenon.

Ibn al-Haytham explains the conditions under which objects will be seen as simple images in binocular vision: they must lie exactly in the same frontal plane or nearly so. Otherwise there will be a disparity, which may go unnoticed if the object is a single color or its texture or pattern are uniform. Ibn al-Haytham brought a rigorously constructed solution to bear on the problem of depth perception – the basis of the theory being the distinction between the case of direct diplopia (the doubling of images of objects located behind the fixation point) and the case of crossed diplopia (the doubling of images of objects located in front of the fixation point). His theories were accessible to scholars through the many texts and commentaries then in circulation, and yet his conception of the fusion of binocular images contrasted markedly with the optical research initiated in early modern Europe, where one notes a tendency to oversimplify the problem. This explains why Ibn al-Haytham's theory remained unchallenged until a new generation of scholars, including Franciscus Aguilonius

(1613), Christoph Scheiner (1619) and Christiaan Huygens (1667), began the problem over.

References

1 Aaen-Stockdale CR. Ibn al-Haytham and psychophysics. *Perception* 2008; 37: 636–638.

2 Aguilonius F. *Opticorum libri VI*. Antwerp: Plantin; 1613: 2, 105–150.

3 Archimedes. *Des corps flottants, Stomachion, La méthode, Le livre des lemmes, Le problème des boeufs*, trans. and ed. C Mugler. Paris: Belles Lettres; 1971: 513.

4 Aristote. *Problèmes*, tome III, ed. P Louis. Paris: Belles Lettres; 1994: 50.

5 Bacon F. *Sylva sylvarum* (1627), ed. J Spedding et al., *The Collected Works of Francis Bacon*. London: Longman and Co; 1857: 2, p. 628.

6 Bacon R. *Perspectiva*, ed. JH Bridges, *The 'Opus majus' of Roger Bacon*. Frankfurt am Main: Minerva; 1964. Ed. DC Lindberg, *Roger Bacon and the Origins of Perspectiva in the Middle Ages*. Oxford: Clarendo Press; 1996 [A: ref_1 pp. 88–91, ref_2 pp. 170–176]

7 Bergdolt K. *Der dritte Kommentar Lorenzo Ghibertis*. Weinheim: VCH, Acta Humaniora; 1988: 294–304, 314–368.

8 Borelli GA. Observations touchant la force inégale des deux yeux. *Journal des Sçavans* 1672; 3: 295–298.

9 Cardano G. *Operum tomus secundus*. Lugduni: JA Huguetan; 1663: 640–641.

10 Carlevaris L. La prospettiva nell'ottica antica: il contributo di Tolomeo. *Disegnare* 1989; 27: 16–29.

11 Clagett M. *Archimedes in the Middle Ages*. Philadelphia: American Philosophical Society; 1964, vol. 1: 668–669; 1984: vol. 5: 297–301.

12 Corbé C, Menu JP, Chaine G. *Traité d'optique physiologique et clinique*. Paris: Doin; 1993: 100–102.

13 Danti E. *Le Due Regole della prospettiva pratica di M. Iacomo Barozzi da Vignola*. Roma: F Zanetti; 1583 [A: pp. 53–55; B: pp. 12, 33].

14 Della Porta GB. *De refractione optices parte libri novem*. Naples: H Salvianus; 1593: Book VI, Cur binis oculis rem unam cernamus [A: pp. 139–148; B: pp. 142–143].

15 Descartes R. *Le Discours de la méthode suivi d'extraits de la Dioptrique, etc.* Leiden: J Maire; 1637; Paris: Flammarion; 1966 [A: p. 139–140; B: p. 149].

16 D'Orleans C. *La Vision parfaite ou Le concours des deux axes de la vision en un seul point de l'objet*. Paris: S Mabre-Cramoisy; 1677: Epitre au Roy.

17 Du Tour EF. Discussion d'une question d'Optique; Addition. *Mémoires de mathématique et de physique présentés à l'Académie royale des Sciences par divers savans* 1760; 3: 514–530; 1763; 4: 499–511.

18 Euclid. *Elements*, vol. 1. *Livres I–IV: Géométrie plane*, ed. B Vitrac. Paris: PUF; 1990: 431–433.

19 Federici Vescovini G. Contributo per la storia della fortuna di Alhazen in Italia: Il volgarizzamento del ms. Vat. 4595 e il 'Commentario Terzo' del Ghiberti. *Rinascimento* 1965; 5: 17–49.

20 Federici Vescovini G. Alhazen vulgarisé: Le 'De li aspecti' d'un manuscrit du Vatican (moitié du XIVe siècle) et le troisième 'Commentaire sur l'optique' de Lorenzo Ghiberti. *Arabic Sciences and Philosophy* 1998; 8: 67–96.

21 Galien C. *De usu partium*, X, 12–13; *Œuvres anatomiques, physiologiques et médicales de Galien*, ed. C Daremberg. Paris: JB Baillière; 1854 [A: pp. 638–645; B: pp. 648].

22 Ghiberti L. *I Commentari*, ed. Ottavio Morisani. Naples: Ricciardi; 1947: 133–136, 139–149.

23 Gorman MJ. Mathematics and modesty in the Society of Jesus: the problems of Christoph Grienberger. In: M Feingold, ed. *The New Science and Jesuit Science. Seventeenth Century Perspective*. Dordrecht: Kluwer; 2003: 41–48.

24 von Helmholtz H. *Optique physiologique*. Leipzig; 1866 and Paris; 1867 [A: p. 880; B: p. 829; C: p. 609; D: p. 912; E: p. 937].

25 Hering E. *Beiträge zur Physiologie*. Leipzig: Engelmann; 1861–1864.

26 Howard IP. Alhazen's neglected discoveries of visual phenomena. *Perception* 1996; 25: 1203–17. Compared with [62] the objective of the two studies are quite different, with Howard devoting three pages to Ibn al-Haytham's study of binocular vision.

27 Ḥunayn Ibn Isḥāq. *The Book on the Ten Treatises on the Eye Ascribed to Hunain ibn Is-hāq (809–877 AD)*, transl. M Meyerhof. Cairo: Government Press; 1928: 26.

28 Huygens C. *Opuscula postuma! quae continent Dioptricam...* Leiden: C Boutesteyn; 1703.

29 Huygens C. Deuxième complément à La Dioptrique. Première partie, 1667. *Oeuvres complètes, tome XIII, Dioptrique*. La Haye: M Nijhoff; 1916: 796.

30 Ibn al-Haytham. *Al-Shukūk ʻalā Batlamyūs (Dubitationes in Ptolemaeum)*, eds. AI Sabra and N Shehaby. Cairo: The National Library Press; 1971: 65. Ibn al-Haytham's critique of Ptolemy has not always been correctly assessed in the literature. [1, 26]

31 Ibn al-Haytham. *Kitāb al-manāẓir*, ed. F Risner, *Opticae thesaurus Alhazeni Arabis libri septem*. Basle: per Episcopios; 1572 (reprint New York: Johnson Reprint Corp.; 1972); ed. AI Sabra, *The Optics of Ibn al-Haytham*, Books 1–3: *On Direct Vision*, 2 vols. London: Warburg Institute; 1989 [A: III, II, 26, ref_1 pp. 81, ref_2 pp. 237–238; B: III, 2, 14, ref_1 pp. 79, ref_2 p. 234; C: ref_1 pp. 16–17; D: ref_1 p. 82, ref_2 p. 239]

32 Kepler J. *Ad Vitellionem paralipomena, quibus astronomiae pars optica traditur*. Frankfurt; 1604: 168.

33 Kheirandish E. *The Arabic Version of Euclid's Optics*. New York-Berlin: Springer; 1999 [A: pp. 80–90; B: pp. 88–99].

34 Le Cat CN. *Traité des Sens*. Amsterdam: J Wetstein; 1744: 204–208.

35 Le Clerc S. *Discours touchant le point de veuë, dans lequel il est prouvé que les choses qu'on voit distinctement, ne sont veuës que d'un oeil*. Paris: T Jolly; 1679.

36 Le Grand Y. *Optique physiologique*, vol. 1: *La dioptrique de l'oeil et sa correction*. Paris: Edition de la Revue d'Optique; 1952 [A: pp. 248–249].

37 Le Grand Y. *Optique physiologique*, vol. 3: *L'espace visuel*, Paris: Edition de la Revue d'Optiqu; 1956 [A: p. 208; B: p. 36; C: p. 215; D: p. 209].

38 Lejeune A. Les recherches de Ptolémée sur la vision binoculaire [A communication delivered in 1957 to the 2nd Benelux Congress on the History of Science]. *Janus* 1958; 47: 79–86 [A: p. 82; B: p. 79].

39 Lejeune A. *L'Optique de Claude Ptolémée dans la version latine d'après l'arabe de l'émir Eugène de Sicile.* Leiden: EJ Brill; 1989 [A: pp. 26–34, 102–118; B: p. 109; C: p. 104; D: p. 103; E: p. 141; F: p. 84].

40 Lindberg DC. Alhazen's theory of vision and its reception in the West. *Isis* 1967; 58: 326–337.

41 Lindberg DC. *Theories of Vision from Al-Kindi to Kepler*. Chicago: University of Chicago Press; 1976: 209–210.

42 Listing JB. *Beitrag zur physiologischen Optik*. Göttingen: W Engelmann; 1845.

43 Long RJ. *Bartholomaeus Anglicus, De proprietatibus rerum, Books 3–4: On the Properties of Soul and Body*. Toronto: Pontifical Institute of of Mediaeval Studies; 1979: 39–45.

44 Müller JP. *Zur Vergleichenden Physiologie des Gesichtssinns*. Leipzig: Cnobloch; 1826.

45 Narducci E. Nota intorno a una traduzione italiana fatta nel secolo decimoquarto del trattato d'ottica d'Alhazen. *Bollettino di bibliografia e di storia delle scienze matematiche e fisiche* 1840; 4: 1–40.

46 Naẓīf M. *al-Ḥasan Ibn al-Haytham*. Cairo: Nūrī Press; 1942-3 [A: vol. 1, pp. 205–217; B: vol. 1, pp. 43–48].

47 Niceron JP. *Mémoires pour servir à l'histoire des hommes illustres dans la République des Lettres*. Paris: Briasson; 1731: vol. 14, 267.

48 Ogle KN. *Researches in Binocular Vision*. Philadelphia-London: Sauders; 1950; 2nd ed 1964.

49 Pansier P. *Collectio ophtalmologica veterum auctorum*, fasc. 7: Ḥunayn Ibn Isḥāq, *Liber de oculi*. Paris: JB Baillière; 1933.

50 Panum PL. *Physiologische Untersuchungen über das Sehen mit zwei Augen*. Kiel: Schwers; 1858.

51 Parme B de. *Questiones super perspectiva communi*, eds. G Federici Vescovini, J Biard. Paris: Vrin; 2009: 178–190.

52 Pecham P. *Perspectiva communis*, ed DC Lindberg, *John Pecham and the Science of Optics*. Madison: The University of Wisconsin Press; 1970 [A: p. 150; B: p. 116].

53 Pecham J. *Tractatus de perspectiva*, ed. DC Lindberg. St Bonaventure: The Franciscan Institute; 1972: 56–57.

54 Pigassou-Albouy R. *Les strabismes*, 1: *Les divergences oculaires*. Paris: Masson; 1991: 27.

55 Pigassou-Albouy R. *Les strabismes*, 2: *Les convergences oculaires*, Paris: Masson; 1992 [A: p. 7; B: p. 9].

56 Platter F. *De corporis humani structura et usu*. Basle: A Frobenium; 1583.

57 Porterfield W. *A Treatise on the Eye. The Manner and Phænomena of Vision*. Edinburgh: G Hamilton and J Balfour; 1759, Vol. 2 [A: pp. 279–328; B: pp. 308–328; C: pp. 314–315].

58 Rashed R. *Entre arithmétique et algèbre*. Paris: Belles Lettres; 1984: 314–315.

59 Rashed R. *Optique et mathématiques*. Aldershot: Variorum; 1992: 235–239.

60 Rashed R. *L'Optique et la catoptrique. Oeuvres philosophiques et scientifiques d'al-Kindī*, vol. 1. Leiden: EJ Brill; 1997

[A: p. 584; B: p. 586; C: pp. 278, 312].
61 Raynaud D. Ibn al-Haytham, Tavoletta binoculare. F Camerota, ed. *Nel segno di Masaccio. L'invenzione della prospettiva*. Firenze: Giunti; 2001: 14.
62 Raynaud D. Ibn al-Haytham sur la vision binoculaire. *Arabic Sciences and Philosophy* 2003; 13: 79–99.
63 Raynaud D. *Studies on Binocular Vi sion. Optics, Vision and Perspective from the Thirteenth to the Seventeenth Centuries*. New York-Berlin: Springer; 2016.
64 Russell G. The anatomy of the eye in ʻAlī Ibn al-ʻAbbās al-Majūsī: A textbook case. In: C Burnett and D Jacquart, eds. *Constantine the African and ʻAlī Ibn al-ʻAbbās al-Majūsī*. Leiden: EJ Brill; 1994: 247–265.
65 Russell G. La naissance de l'optique physiologique. In: R Rashed, ed. *Histoire des Sciences Arabes*. Paris: Seuil; 1997: 2, 319–354.
66 Sabra AI. Ibn al-Haytham's criticism of Ptolemy's Optics. *Journal of the History of Philosophy* 1966; 4: 145–149.
67 Sabra AI. The astronomical origin of Ibn al-Haytham's concept of experiment. *Actes du XIIe Congrès international d'his toire des sciences*. Paris: A Blanchard; 1971: vol. IIIA, 133–136.
68 Sabra AI. Sensation and inference in Al hazen's theory of visual perception. In: PK Machamer and RG Turnbull, eds. *Studies in Perception*. Columbus: Ohio State Uni versity Press; 1978: 164.
69 Sabra AI. *The Optics of Ibn al-Haytham, Books I–III: On Direct Vision*. London: Warburg Institute; 1989 [A: vol. 2, p. 125; B: vol. 2, pp. 45–51; C: vol. 2, pp. 123–124; D: vol. 2, p. lxiv; E: vol. 1, pp. 228–229].
70 Saraux H and Biais B. *Physiologie ocu laire*. Paris; 1973, 2nd ed. 1983 [A: pp. 390–391; B: p. 336; C: pp. 393–394].
71 Scheiner C. *Oculus, hoc est fundamentum opticum*... Inspruck: D Agricola; 1619, London: J Flesher; 1652: 32–49.
72 Schramm M. *Ibn al-Haythams Weg zur Physik*. Wiesbaden: F Steiner; 1963.
73 Simon G. *Le Regard, l'être et l'apparence dans l'optique de l'Antiquité*. Paris: Seuil; 1988: 131–132.
74 Simon G. La psychologie de la vision chez Ptolémée et Ibn al-Haytham. In: A Hasnawi, A Elamrani-Jamal and M Aouad, eds. *Perspectives arabes et médiévales sur la tradition scientifique et philosophique grecque*. Louvain: Peeters; 1997: 189–190.
75 Smith AM. Goldstein BR. The medieval Hebrew and Italian versions of Ibn Muʻadh's 'On twilight and the rising of clouds'. *Nuncius* 1993; 8: 633–639.
76 Smith AM. *Ptolemy's Theory of Visual Perception*. Philadelphia: American Philosophical Society; 1996: 60.
77 Smith AM. *Alhacen's Theory of Visual Perception*. Philadelphia: American Philosophical Society; 2001 [A: p. 266; B: pp. lxxx-cxi; C: pp. ix, xx–xxi; D: pp. 256–257; E: p. 245; F: p. 286; G: p. 247; H: p. 252; I: p. 268; J: p. 269; K: p. 259; L: p. 261; M: p. 262; N: pp. 260–261].
78 Spielmann A. *Les strabismes*. Paris: Masson; 1991 [A: p. 116; B: p. 117].
79 Stidwill D, Fletcher R. *Normal Binocular Vision*. Oxford: Wiley-Blackwell; 2011 [A: p. 75; B: pp. 61–62; C: pp. 29–30].
80 Theisen WR. Liber de visu. *Mediaeval Studies* 1979; 41: 78–80.
81 Thomas C. La physiologie de la vision binoculaire. *Archives d'Ophtalmologie (Paris)* 1971; 31: 191–192.
82 Vieth GUA. Ueber die Richtung der Augen. *Gilbert's Annalen der Physik* 1818; 58: 233–253.
83 Vinci L da. *Tratatto della pittura*, §115: Perche la pittura non può mai parere spicata come le cose naturali, and §811: Di prospettiva, ed. A Borzelli. Lanciano: Carabba; 1924: 76, 401.
84 Wade NJ. Descriptions of visual phenomena from Aristotle to Wheatstone. *Perception* 1996; 25: 1137–75.
85 Wade NJ. Early studies of eye dominances. *Laterality* 1998; 3: 104.
86 Wade NJ, Ono H. Early studies of binocular and stereoscopic vision. *Japanese Psychological Research* 2012; 54: 54–70.
87 Wheatstone C. Contributions to the physiology of vision. – Part the First. On some remarkable, and hitherto unobserved, phænomena of binocular vision. *Philosophical Transactions of the Royal Society of London* 1838; 128: 371–394.
88 Witelo, *Opticae libri X*, ed. F Risner. Basle: per Episcopios; 1572 (reprint New York: Johnson Reprint Corp.; 1972: 98–108).